A SYSTEMS ANALYSIS OF A MARITIME TRANSPORTATION NETWORK

by

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A Thesis Submitted to the Faculty of the

DEPARTMENT OF SYSTEMS ENGINEERING

In Partial Fulfillment of the Requirements
For the Degree of

MASTER OF SCIENCE

In the Graduate College

THE UNIVERSITY OF ARIZONA

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STATEMENT BY AUTHOR

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ACKNOWLEDGMENTS

The author wishes to express his sincere gratitude to Dr. Wayne Wymore and Dr. Lucien Duckstein for their advice, guidance, and great patience during the preparation of this thesis.

He is also grateful to Dr. Robert Baker for his helpful criticisms and for his participation on the committee.

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ABSTRACT

The objective of this thesis is to analyze a Maritime Transportation Network with a strict system theoretic approach in hopes that the analysis will yield valuable information that may be used to develop a solution to the scheduling problem. This study starts with a clean slate: no assumptions are made about what should be an optimal scheduling policy. The total number of assumptions is kept to a minimum in order to insure an unbiased decision as to the best technique to use to solve the problem. Emphasis is placed on the interaction between man and the computer; the computer does the evaluations and the man makes the judgmental decisions based on his experience and insight. The solution technique is programmed in a basic version of FORTRAN IV to insure machine independence.

CHAPTER 1

DEFINITION OF THE PROBLEM

Introduction

The transportation industry is one of the nation's largest and most important. Yet this industry has a problem that has resisted solution for many years despite the expenditure of much time and effort toward its solution. The specific problem is one of optimally scheduling the flow of goods from a set of origins to a set of destinations on a dynamic time scale. Thus the action of the system at time "t" will influence future actions that may occur at time "t + n."

Among the many advances that have occurred in the field of operations research is a technique for solving the classical form of the scheduling problem (Sasieni, Yaspan and Friedman 1966). This classical problem, sometimes referred to as the Hitchcock problem, does not, however, consider the dynamic time element, which is the parameter that causes most of the difficulties in a transportation network (Hogan 1967). The scheduling problem referred to above is a problem of the entire industry whether it be a train system, airplane system or a system of vessels. The problems discussed in this thesis will be limited to those encountered in a maritime transportation system with ships providing the movement of goods.

The maritime shipping industry like other industries has spent much time and money in developing computer systems to aid in their operational problems. Many problems such as accounting and inventory control have lent themselves very readily to complete solution by operations research techniques and computer programs, while others have proven very difficult problems for which to find an optimization technique to solve completely the difficulties. In general, it seems that the most difficult problems to solve are those in which human judgment, learning, and experience are the basis for a solution. With the advent of the high-speed digital computer it has been very difficult for scientists and engineers to admit that there may be some problems that can be solved best by a human using his judgment, insight, and experience, merely aided by a computer doing the routine calculations. This author feels that this attitude has been costly in the past and must be avoided in the future. The attitude taken in this thesis will be to learn a maximum amount of information about the maritime transportation scheduling problem in hopes that this knowledge will aid in finding some solution technique, whether it be a solution technique that when programmed on a computer will yield the optimal solution, or whether it be a solution technique that simply uses the computer as a computational tool enhancing human judgment. The method of obtaining this valuable information will be to use general system theory (Wymore 1967) as an analysis tool that will allow the system to be modeled realistically and yet mathematically.

The Scheduling Problem

The problem can be stated as follows. Let there be N origins, M destinations, NSHIP ships and NCARGO cargoes. Let each origin I have a total of G(I) cargoes, where $\sum_{I=1}^{N} G(I) = \text{NCARGO}$ and where each cargo J is of type U(J), of amount S(J), has port of origin O(J), port of destination D(J), must be moved between dates E(J) and L(J), and must arrive at port D(J) by date F(J). Let each destination q have a demand d(q). Let each ship K be capable of carrying cargo of types Z(K), and have a capacity r(K, J) when carrying the Jth cargo. Let t(K, O(J),D(J)) be the time for the Kth vessel to travel between the ports of origin and destination when carrying cargo J. Let P(K, J) be the profit incurred for the corporation by the Kth vessel carrying the Jth cargo. The solution to this problem will be a matrix A(J, K), with A(J, K) = 1 if the Kth vessel is assigned to carry the Jth cargo and A(J, K) = 0 otherwise. Let h(J, K) be the date on which the Jth cargo is picked up by the Kth vessel.

Mathematically then, the problem is to maximize

$$X = \sum_{K=1}^{\text{NSHIP}} \sum_{J=1}^{\text{NCARGO}} P(K,J)*A(J,K), \quad \text{subject to}$$

$$U(J) \varepsilon Z(K), \forall J,K \ A(J,K) = 1,$$

$$S(J) \leq r(K,J) \forall J,K \ A(J,K) = 1,$$

$$E(j) \leq h(J,K) \leq L(j) \forall j,K \ A(J,K) = 1,$$

$$h(J,K) + t(K,O(J),D(J)) \leq F(J) \forall K,J \ A(J,K) = 1,$$

$$\sum_{J=1}^{NCARGO} S(J) \leq \sum_{q=1}^{M} d(q)$$

Stated in words the problem is to maximize the profit for the corporation by assigning ships to move the cargoes. A feasible solution must satisfy the constraints defined above, i.e., if ship K is assigned to carry cargo J then the type of cargo J must be such that ship K can carry it. Also the capacity of ship K must be large enough to load cargo J. Ship K must be able to pick up cargo J within the specified time periods, and the pick-up date and travel time must allow ship K to deliver cargo J to its destination on time.

The crux of the problem is to choose the A(J,K) matrix from the set of all NCARGO X NSHIP matrices so as to maximize the objective function. Not only are there a great many alternatives to choose from but the effect of each assignment on the total movement of cargo must be considered. Thus an assignments that appears to be a good one in time period "t" may cause a very bad assignment to be made in time period "t+n."

Previous Studies

Three previous studies of this problem have been carried out by graduate students in Systems Engineering at The University of Arizona. The first of the studies (Preston 1966) produced background and historical information concerning the scheduling problem of a particular shipping company. The solution technique developed in the Preston thesis was a simulation approach. The simulation idea is a sound one but in this particular case the finished product was extremely difficult to use. Every corporate policy had to be programmed so as to schedule the vessels according to current corporate policies. Thus if a change in policy was to be evaluated the program had to be changed so as to schedule vessels according to the new policy. It appears as if this fact alone detracts greatly from one advantage of a simulation: to be able to evaluate real world changes on a computer at a low cost. The Preston thesis, however, did lay important ground work for later research into the subject.

The second research effort into the problem (Hogan 1967) resulted in an heuristic computer program to solve the problem the way a human would if he were doing it by hand. Many supposedly optimum assumptions were made and became part of the model. Also the model had to be supplied with decision rules for the assignment of vessels to cargo. These decision rules had to encompass every possible circumstance that could arise and then dictate the decision to be made.

The above two studies although very helpful in defining the transportation scheduling problem resulted in solution techniques that when programmed on a computer were trying to compete with the human who has many years experience in the scheduling of vessels. The result then of trying to have a computer duplicate human judgments is two solution techniques that although useful do not yield an optimal solution and are very difficult to use.

The most recent work done on this problem at the University of Arizona (Xenos 1968) obtained the best results and used more of a system theoretic approach. The approach was to decompose the problem into a set of several indirect problems, solve these indirect problems using linear programming and dynamic programming techniques and then connect the various component solutions into a solution of the original problem. This technique again did not arrive at an optimal solution to the problem, but was very close to one in the limited number of experiments that were conducted with the model. However, the worth of a solution technique unfortunately is not only measured by the accuracy of the solution obtained but also by the acceptance of it by people that must use it. Once again the derived solution technique proved to be very difficult to use in a real life situation and therefore was not accepted by the industry (Tuchscherer 1971).

Since the above models did not find an optimal solution to the scheduling problem and appear not to be feasible to use in a real life environment some reasons for this must be found.

1. The computer programs that represented the above solution techniques examine each ship individually to obtain an optimal schedule for some scheduling time period. The actual schedule produced is not necessarily an optimal one, and even if it were there is no guarantee that combining optimal ship schedules will produce a total optimal schedule for the entire fleet of ships. In other words, the effect of each vessel on the other vessels must be taken into account when trying to derive an optimal solution for an entire fleet of ships.

2. Too many "supposedly optimal" policies were built into the models. Thus the transportation system was never modeled as a separate

entity. What was modeled was the way the system operated when it was controlled by certain policies.

It is this author's feeling that this reasoning drastically affected the above efforts. If, in fact, certain policies are the best then any model that is to produce an optimal schedule should not be forced to follow a set of policies. If they are optimal, the logical sequence of events in the model should prove them so. That is, a model should produce results that may be described in terms of optimal policies. Therefore, it is important in the development of a solution technique to model first only the basic actions of the system, then using this model, policies may be tested and only then may the decision be made as to what policies are optimal. In this manner the predetermined policies will not become part of the basic model and thus much confusion will be avoided.

Research other than at the University of Arizona has also been done. One of the earliest research efforts in the vessel scheduling problem concerned the problem of minimizing the number of vessels needed to meet a fixed schedule (Dantzig and Fulkerson 1954). This problem was formulated into a linear programming problem and solved using the simplex method. This model assumed however that all cargoes must be delivered on time and thus was not very realistic. Further work resulted in a technique for solving the same problem assuming that some cargoes could not be delivered and therefore must be canceled (Bellmore, Bennington and Lubore 1968). The Bellmore work shows that this problem becomes one of solving the transshipment problem.

Whereas the above research work is useful, it is not applicable to the problem faced by commercial shipping companies. These companies have a fixed fleet size and are looking for cargo to keep their vessels active.

Much effort has been spent formulating scheduling problems into a linear programming problem (Dantzig 1963). Obtaining a solution to the LP formulated problem has been very troublesome in that computer time and memory requirements have been prohibitive. Research at the Royal Institute of Technology, Stockholm, Sweden, has produced an algorithm that uses the Dantzig-Wolf decomposition model as a basis. In the initial report of this research (Appelgren 1969) all but about two percent of the experimental problems were solved using a dynamic programming solution. The two percent that were not solved yielded fractional solutions. Thus the LP formulation of the problem and solution cannot guarantee an integer solution and therefore a feasible one. Later work however (Appelgren 1971) has arrived at a suitable integer programming method to solve the fractional cases. The method developed uses the fractional solution as a bound and the branching is made with the objective of maximizing the probability that the solution of the revised LP problems become integer. Much experimentation has been done using the above technique but using small system configurations, i.e., 10 ships and 15 cargoes. Therefore no data are available concerning the cost of finding the optimal solution for a more realistic problem. Although the computer solutions obtained from the above algorithm are optimal, the shipping company that uses it does not solely rely on it.

The computer solutions are subject to manual revisions because of details concerning exact arrival and departure times and split load or discharging. Therefore these computer schedules are regarded more as very good tentative schedules rather than as a final product. Their main advantage is that some of the more tedious work of the scheduling officer is now done by computer. This confirms this author's feeling that human judgment and experience must take the major role in finding a good solution technique to this problem.

Other work has been done on the scheduling problem in areas other than vessel scheduling. Whereas this work cannot always be applied to vessel scheduling some parallels may be obvious. One such area is that of scheduling delivery of health services to rural areas such as Indian reservations (Klunk 1970). This problem as described in the Klunk thesis may be characterized in the following manner:

1. Let there exist a number of villages where health commitments exist.

2. Let there be a certain number of health teams available that can fulfill these commitments.

3. Let the road network and transportation times between villages be known.

The problem then is to minimize the cost of fulfilling the commitments with the resources and time that are available. A parallel to the vessel scheduling problem may be obtained if each health commitment is thought of as a cargo that must be delivered at a specified time. Let each health team represent a vessel that can move the cargo. Therefore a solution to both problems is a schedule of routes for teams or vessels that can fulfill the commitments that exist at the various villages or ports. The Klunk thesis develops a technique that is an iterative process using a combination of a branch and bound algorithm and a heuristic elimination of unattractive routes or schedules. This solution technique does not generate an optimal solution but does provide valuable information that may be used by a human to find the optimal solution.

Thesis Outline

The first step will be to develop a realistic but rigorous model of a maritime transportation network using general system theory as a building tool. After deriving the model it will then be implemented via a computer program. In this manner experiments with the model may be conducted and experience may be gained with the system. After experiments have been conducted with the basic system the question as to the best method of solution to the scheduling problem may be answered. A solution technique will be developed and implemented. Finally, conclusions will be made as to the usefulness of the solution technique in the real world.

CHAPTER 2

DEVELOPMENT OF THE MODEL

The models developed here will be described using system theoretic concepts developed by Wymore (1967). Some of the notation used may be unfamiliar to the reader; therefore, a brief description of it will be given here.

System Theoretic Concepts and Notation

Let $\mathcal{F}(A,D)$ denote the set of all functions defined on a set A with values in a set D and let $f \in \mathcal{F}(A,D)$. The domain of the function f will be denoted $\Delta(f)$, and the range of f will be denoted $\Gamma(f)$.

Let I denote the set of all integers, I^+ denote the set of positive integers, I^{++} denote the non-negative integers and $I[n,m]$ denote the closed interval of integers from n to m. Let R denote the set of real numbers, R^+ the positive reals, R^{++} the non-negative reals, and $R[s,t]$ denote the closed interval of real numbers from s to t.

A system must have a set F of admissible input functions. If f, $g \in F$ then $(f \rightarrow r)$ must be in F and $(f|g)$ must also be in F, where $(f \rightarrow r)$ is the translation of f by r time units, and $(f|g)$ is the segmentation of f and g. By definition, then

$$(f \rightarrow r)(t) = f(r+t) \ ,$$

and

$$\begin{aligned} (f|g)(t) &= f(t) \ , && \text{if } t<0, \\ &= g(t) \ , && \text{if } t \geq 0 \ . \end{aligned}$$

Let $\mathcal{G}(\{c_p : p \varepsilon P\})$ denote the set of all step functions defined on R with values in a set P, with a finite number of steps constant in each interval between steps.

If $A_1, \ldots, A_n$ are sets let $A_1 X \ldots XA_n = \{(a_1, \ldots, a_n) : a_i \varepsilon A_i \forall i \varepsilon I\ [1,n]\}$; let the projection function Π_i be defined as follows: Π_i is a function defined on $A_1 X \ldots XA_n$ with values in A_i as follows:

$$\Pi_i(A) = a_i \forall (a_1, \ldots, a_n) \varepsilon A_1 X \ldots XA_n \ .$$

The systems that will be presented below belong to the discrete class of systems. A discrete system may be defined by a sixtuple $Z = \{S,P,F,M,T,\sigma\}$, where S is a set not empty that represents the set of states of the system; P is a set not empty that represents the set of all possible input values to the system; F is the set of admissible input functions defined on R with values in P, i.e., $F \varepsilon \mathcal{F}(R,P)$; M is the set of all transition functions defined on S with values in S; T is the time scale of the system which in the discrete case must be I^{++}; and σ is a mapping defined on FXT and mapped onto M. This function describes the manner in which the components of S change and must have the following properties:

1. The identity mapping ω must be in M and if $f \varepsilon F$ then $\sigma(f,0) = \omega$,
2. If $f \varepsilon F$ and $s \varepsilon T$ such that $s+t \varepsilon T$ then $(\sigma(f \to s,t)(\sigma(f,s)) = \sigma(f,s+t)$,
3. If $f \varepsilon F$ and $g \varepsilon F$, $s \varepsilon T$ and $f(t) = g(t) \forall t \varepsilon R[0,s)$ then $\sigma(f,s) = \sigma(g,s)$.

Decomposition of a Maritime Transportation Network

A maritime transportation network may be decomposed into three basic entities; ships, ports, and cargo. This section will be devoted to modeling each of these entities using general system theory as the tool to mathematically and realistically describe their action.

Let SHIPS = {SHIP(L):LεI[1,NSHIP]}, be a set that represents the fleet of vessels that are available to be scheduled. Let PORTS = {PORT(J): JεI[1,NPORT]} be a set that represents all the ports that are to be considered in the network and let CARGOS = {CARGO(K) :KεI[1,NCARGO]}, be a set that represents all the cargoes that must be moved.

The first objective will be to model one element from each of these sets as a system to examine its behavior, and then to couple the three subsystems together forming a system that describes the total transportation network. Much care will be taken in the initial modeling phase for it is this decomposition and modeling that is the basis for later optimization. By first modeling these entities independently of each other, and then coupling the systems together, a subsystem's particular role and its importance to the total system will be made very clear. As the modeling proceeds, unique features of each of the systems will be pointed out in hopes that this will give a much greater insight into the operation of the total system.

The Cargo Model

The cargo model will be described first for it is the simplest and therefore will serve best as an introduction to systems models.

Let CARGOS be the set as defined above, but assume that

$$\text{CARGOS} \subseteq \text{OPORT} \times \text{DPORT} \times \text{TYPE} \times \text{AMOUNT} \times \text{EDATE} \times \text{LDATE} \times \text{DUEDATE} \times \text{STATUS} \times \text{PDATE} \times \text{DDATE} \times \text{ISHIP},$$

and

$$\text{OPORT} = I[1,\text{NPORT}] , \quad \text{DPORT} = I[1,\text{NPORT}] ,$$

$$\text{TYPE} = \{\text{OIL},\text{ORE},\text{COAL}\} ,$$

$$\text{AMOUNT} = I^{++} , \quad \text{LDATE} = I^{++} ,$$

$$\text{EDATE} = I^{++} , \text{DUEDATE} = I^{++} , \text{PDATE} = I^{++} , \text{DDATE} = I^{++} ,$$

$$\text{STATUS} = \{\text{READY},\text{NOTREADY},\text{PICKEDUP},\text{NOTASSIGNED}\} ,$$

$$\text{ISHIP} = I[1,\text{NSHIP}] .$$

Then each CARGO (I)εCARGOS is of the form:

$$\text{CARGO (I)} = (\text{OPORT(I)}, \text{DPORT(I)}, \text{TYPE(I)}, \text{AMOUNT(I)}, \text{EDATE(I)}, \text{LDATE(I)}, \text{DUEDATE(I)}, \text{STATUS(I)}, \text{PDATE(I)}, \text{DDATE(I)}, \text{ISHIP(I)}) ,$$

where OPORT(I) and DPORT(I) are the port of origin and the port of destination of the Ith cargo with OPORT(I) $\neq$ DPORT(I) and are elements of the set OPORT and DPORT, respectively. TYPE(I) represents the type of the Ith cargo and is an element of the set TYPE. AMOUNT(I) is the amount, in tons, of the Ith cargo and is an element of the set AMOUNT. EDATE(I) and LDATE(I) are the early and late pick-up dates of the Ith cargo and are elements of the sets EDATE and LDATE, respectively. DUEDATE(I) is the date by which the Ith cargo must be delivered to port of destination, where DUEDATE(I)εDUEDATE. The STATUS(I) component represents whether the cargo is ready to be picked up, not ready to be picked up, already has been picked up at a particular time, or has not been scheduled to be picked up during the time period under consideration and STATUS(I)εSTATUS.

PDATE(I) is the date on which the Ith cargo was picked up while DDATE(I) is the date on which the Ith cargo is to be delivered to its port of destination where PDATE(I)εPDATE and DDATE(I)εDDATE. These components will have a value of zero initially. ISHIP(I) is the identification of the ship that carries the Ith cargo, where ISHIP(I)εISHIP.

Let $Z_{CARGO(I)} = \{S_{CARGO(I)}, P_{CARGO(I)}, F_{CARGO(I)}, M_{CARGO(I)}, T_{CARGO(I)}, \sigma_{CARGO(I)}\}$, be a system that represents the Ith cargo, where

$$S_{CARGO(I)} = (\{OPORT(I)\} \times \{DPORT(I)\} \times \{TYPE(I)\} \times \{AMOUNT(I)\} \times \{EDATE(I)\} \times \{LDATE(I)\} \times \{DUEDATE(I)\} \times STATUS \times PDATE \times DDATE \times ISHIP) ,$$

$$P_{CARGO(I)} = DATE \times SHIPS ,$$

$$F_{CARGO(I)} = \mathcal{G}(\{c_p : p \varepsilon P_{CARGO(I)}\}) ,$$

$$M_{CARGO(I)} = \Gamma(\sigma_{CARGO(I)}) ,$$

$$T_{CARGO(I)} = I^{++} ,$$

$$\sigma_{CARGO(I)}(f,t)(x) = \sigma_{CARGO(I)}(c_{f(t-1)},1)\ \sigma_{CARGO(I)}(f,t-1)(x) ,$$
$$= x \text{ if } t = 0 .$$

Cargo State Transitions

In order to complete the model the state transition function $\sigma_{CARGO(I)}$ must be displayed. To display $\sigma_{CARGO(I)}$ is to describe how each component of $S_{CARGO(I)}$ changes under all possible conditions. For this particular model it is a fairly simple task, but in general, as will be seen later, it is this graph of the transition function that is the crux of modeling.

The first seven components of the state set are non-dynamic, their values are established when the cargo parcel is created; therefore:

$$\Pi_j(\sigma_{CARGO(I)}(c_p,1)(x) = \Pi_j(x) \ \forall j \ I[1,7] .$$

The other components of S_{CARGO} change as follows

$$\Pi(\text{STATUS}) \ \sigma_{CARGO(I)}(c_p,1)(x)$$

$= \text{NOTREADY}$ if $\Pi(\text{DATE})(p) \leq \Pi(\text{EDATE}(I)(x))$

$= \text{READY}$ if $\Pi(\text{DATE})(p) \geq \Pi(\text{EDATE}(I)(x))$,

and $\Pi_1(\text{SHIP}(J)(p)) \neq \text{LOADING}$,

and $\Pi_2(\text{SHIP}(J)(p)) \neq I \ni J \varepsilon I[1,\text{NSHIP}]$,

and $\text{SHIP}(J) \varepsilon \text{SHIPS}$.

$= \text{PICKEDUP}$ if $\Pi_1(\text{SHIP}(J)(p)) = \text{LOADING}$,

and $\Pi_2(\text{SHIP}(J)(p)) = I \ni J \varepsilon I[1,\text{NSHIP}]$,

and $\text{SHIP}(J)(p) \varepsilon \text{SHIPS}$,

and $\Pi(\text{DATE})(p) \geq \Pi(\text{EDATE}(I)(x))$.

$= \text{NOTASSIGNED}$ if $\Pi_2(\text{SHIP}(J)(p)) \neq I \ni \forall J \varepsilon I[1,\text{NSHIP}]$,

where $\text{SHIP}(J) \varepsilon \text{SHIPS}$.

$\Pi(\text{PDATE})\sigma_{\text{CARGO}(I)}(c_p,1)(x)$

$= \Pi(\text{DATE}(p))$ if $\Pi_1(\text{SHIP}(J))(p)) = \text{LOADING}$,

and $\Pi_2(\text{SHIP}(J)(p)) = I \ni I \varepsilon I[1,\text{NCARGO}]$,

and $\Pi_3(\text{SHIP}(J)(p)) = 0$, $\ni \text{SHIP}(J) \varepsilon \text{SHIPS}$,

and $\Pi(\text{STATUS}(I)(x)) = \text{READY}$,

$= 0$ otherwise .

$\Pi(\text{DDATE})\ \sigma_{\text{CARGO}(I)}(c_p,1)(x)$

$= \Pi(\text{DATE}(p))$ if $\Pi_1(\text{SHIP}(J)) = \text{UNLOADING}$,,

and $\Pi_2(\text{SHIP}(J)(p)) = I$,

and $\Pi_3(\text{SHIP}(J)(p)) = 0$,

and $\Pi(\text{STATUS}(I) = \text{PICKEDUP}$,

and $\text{SHIP}(J) \varepsilon \text{SHIPS} \ni J \varepsilon I[1,\text{NSHIP}]$,

$= 0$ otherwise.

$\Pi(\text{ISHIP})\ \sigma_{\text{CARGO}(I)}(c_p,1)\ (x)$

$= J \ni J \varepsilon I[1,\text{NSHIP})$ if $\Pi_1(\text{SHIP}(J)(p)) = \text{LOADING}$,

and $\Pi_2(\text{SHIP}(J)(p)) = I$,

and $\Pi_3(\text{SHIP}(J)(p)) = 0$,

and $\Pi(\text{STATUS}(I)(x)) = \text{READY}$,

where $\text{SHIP}(J) \varepsilon \text{SHIPS}$.

In words, if the current date is less than the earliest possible pickup date the cargo has a NOTREADY status; if the current date is equal to or greater than the earliest possible pickup date then the cargo has a status of READY; if the cargo has been picked up then the status is PICKEDUP; if the cargo has not been assigned a ship and all the ships are done carrying cargo then the status is NOTASSIGNED. When the cargo is picked up, the current date and the ship identification are recorded and become part of the state set. When cargo is delivered the current date also becomes part of the state set. These dates and the ship number will be used later when the schedule is evaluated.

The system $Z_{CARGO(I)}$ has two input components, DATE and SHIPS. The DATE component is equal to the total number of time units that the system has been running. The SHIPS component represents the state of all vessels in the system. The system $Z_{SHIP(J)}$ will be described later in this thesis.

The system $Z_{CARGO(I)}$ will also have an output function. By definition this function $\zeta_{CARGO(I)}$ is defined on $S_{CARGO(I)}$ with values in a set $Q_{CARGO(I)}$. This output function will be described in more detail later.

In keeping with the premise set forth earlier, that too many assumptions disguise the actual system operation to a point where optimization may be impossible, a check as to the assumptions needed to develop the above model is in order.

It was assumed that for each cargo parcel an early and late pickup date could be established, as well as a date by which the cargo must be delivered. These assumptions are in fact valid and are an aid in modeling reality rather than a hindrance. In today's shipping industry, goods to be shipped are contracted for in amounts much larger than can be transported by one vessel making one trip. Therefore, these large contracts must be decomposed into smaller parcels that are referred to as cargo parcels or cargoes in this thesis. Contracts, more often than not, specify a particular time by which all cargo must be picked up and delivered. A penalty clause that may be invoked if the deadline is not met is usually included. Therefore, if a shipping company is to try to meet its contractual demands some sort of time table for pickup delivery must be set up. It is for this reason that the establishment of the EDATE, LDATE, AND DUEDATE times is very consistent with reality (Xenos 1968). It may seem that inherently it has been assumed that the best policy for a shipping company is to pick up and deliver all its cargo within the specified time limits. This, however, is not the case, as whether or not cargoes are picked up and delivered on time is a function of the schedule and nothing else. Hence, once a schedule is established, if no changes are made, pickup and delivery dates are fixed at that point in time. The system that is described here, however, does not do the scheduling but rather will evaluate the worth of a particular schedule. A schematic of the system $Z_{CARGO(I)}$ appears in Figure 1.

The Ship Model

The next step in the development of the total system is to model a ship as a system. Let SHIPS be the set as defined above but assume that SHIPS $\subseteq$ SACTION X CARGOS X RELTIME X OPORT X DPORT X CAPACITY X STYPES X SPEEDS X ULTIMES X LTIMES X ITRIP, where

SACTION = {SAILING,BALLAST,LOADING,UNLOADING,FREE,WAITING}

ITRIP = RELTIME = I^{++} ,

OPORT = I[1,NPORT] ; DPORT = I[1,NPORT] ,

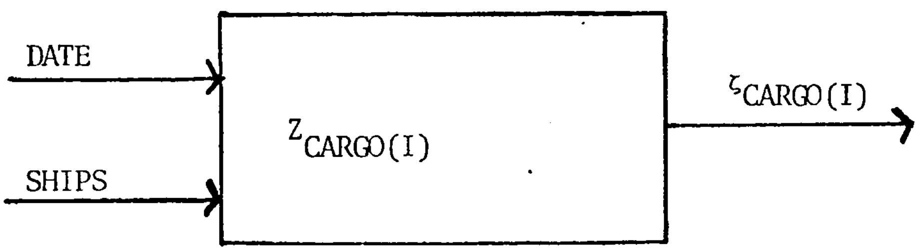

Figure 1. The cargo system.

$$\text{CAPACITY} = \mathcal{F}(\Pi_3(\text{CARGOS}),\ I^{++}) \ ,$$

$$\text{STYPES} = \text{SUBSETS}(\{\text{OIL},\text{ORE},\text{COAL}\}) \ ,$$

$$\text{SPEEDS} = \mathcal{F}(\text{SACTION},\ R^{++}) \ ,$$

$$\text{ULTIMES} = \mathcal{F}(\Pi_3(\text{CARGOS}) \times \text{DPORT},\ I^{++}) \ ,$$

$$\text{LTIMES} = \mathcal{F}(\Pi_3(\text{CARGOS}) \times \text{OPORT},\ I^{++}) \ .$$

Then each SHIP(J)εSHIPS is of the form:

$$\text{SHIP(J)} = (\text{SACTION(J)},\ \text{CARGO(J)},\ \text{RELTIME(J)},\ \text{OPORT(J)},$$
$$\text{CAPACITY(J)},\ \text{STYPES(J)},\ \text{SPEEDS(J)},\ \text{ULTIMES(J)},\ \text{LTIMES(J))},$$

where SACTION(J) is the current action of the vessel and SACTION(J)εSACTION. CARGO(J) is the cargo that the vessel is carrying or will be carrying on its next loaded trip, where CARGO(J)εCARGOS. RELTIME(J) is the time units remaining until the vessel completes its current action and RELTIME(J)εRELTIME. OPORT(J) and DPORT(J) are the vessel's port of origin and port of destination, respectively, for a given trip where OPORT(J)εOPORT, and DPORT(J)εDPORT. CAPACITY(J) is the maximum amount of cargo that a vessel may carry, where CAPACITY(J)εCAPACITY. STYPES(J) is the set of cargo types that the vessel may carry and is an element of the set STYPES. SPEEDS(J) is the different speeds at which the vessel is capable of traveling and is an element of SPEEDS. ULTIMES(J) and LTIMES(J) are the unloading and loading times for the vessels where ULTIMES(J) and LTIMES(J) are elements of ULTIMES and LTIMES, respectively. A more detailed discussion of the above parameters is in order.

Each vessel is capable of carrying certain amounts of cargo, certain types of cargo, and is capable of traveling at certain speeds. These are physical properties of a vessel that are built into it at construction time. In a physical sense these are non-dynamic parameters; they are the parameters that determine the feasibility of schedules. As defined above the capacity of a vessel is a function of the type of cargo that is being carried. This is due to the different densities of cargoes (Preston 1966). There are, however, other parameters that influence how much cargo a vessel is capable of carrying on a given trip. Draft limitations at the port of origin or destination, or canal restrictions may dictate that a vessel carry less than its actual maximum capacity. Therefore, on any given trip the amount of cargo that a vessel may carry is the minimum of its capacity, any draft limitations, or any canal restrictions.

In this model a vessel will be capable of traveling at two speeds, a ballast speed and a loaded speed. It was found that by using these two speeds accurate estimates of time required for a ship to make various trips could be made (Preston 1966). Knowledge of a ship's speed is necessary to evaluate the cost of executing a schedule.

The loading and unloading times are a function of the type of cargo that is being moved and the port facilities that are being used. Any shipping company would have the necessary information recorded so that the above times could be estimated for all ships on all trips.

The dynamic parameters of a vessel are SACTION(J), RELTIME(J), CARGO(J), OPORT(J), DPORT(J), ITRIP(J). The actions that a vessel is allowed to have are self explanatory except for the action of FREE. This is the action of a vessel that has completed delivery of all of its assigned cargo for a given scheduling period. The time that any vessel spends in the FREE status may be used as a measure of a schedule's usefulness. This will be discussed in more detail later in the thesis. The CARGO(J) component is the cargo that the vessel is carrying or is traveling ballast to pick up. The RELTIME(J) component is the amount of time units remaining until the current action SACTION(J) is completed. The ITRIP(J) component is the number of the assignment that the Jth vessel is currently executing. The component DPORT(J) will always be equal to the vessel's next port of destination. For example, when a vessel enters a port to unload a cargo its next cargo assignment is found and DPORT(J) is set equal to that cargo's port of origin. This is done so as to aid in the process of looking ahead in time, which is vital to any algorithm that will make assignments. A typical set of values for these dynamic components may be:

BALLAST 14 2 3 4 1 .

Thus this vessel is traveling unloaded from port 3 to port 4 to pick up cargo 14 and there are two time units remaining in the vessel's first trip.

Input to the system will be a 4-tuple, {ASSIGNMENTS X CARGOS X PORTS X DISTANCE}, where ASSIGNMENTS$\varepsilon\mathcal{F}$(I[1,NCARGO] X I[1,NSHIP], I^{++}), CARGOS is the set as defined above, PORTS represents the set of all ports, and DISTANCE$\varepsilon\mathcal{F}$(I[1,NPORT[X I[1,NPORT], I^{++}). Thus, ASSIGNMENTS is an NCARGO X NSHIP matrix where the (I,J)th element has a value of one if the

Jth vessel has been assigned to carry the Ith cargo or a value of zero if the Jth vessel has not been assigned to carry the Ith cargo. Similarly DISTANCE is an NPORT X NPORT matrix where the (I,J)th element is the distance between port I and port J.

With the above parameters defined the system $Z_{SHIP(J)}$ can now be defined. Let

$$Z_{SHIP(J)} = \{S_{SHIP(J)}, P_{SHIP(J)}, F_{SHIP(J)}, M_{SHIP(J)}, T_{SHIP(J)}, \sigma_{SHIP(J)}\} ,$$

where

$$S_{SHIP(J)} = (\{SACTION\} \times \{CARGO(J)\} \times \{RELTIME(J)\} \times \{OPORT(J)\} \times \{DPORT(J)\} \times \{ITRIP(J)\} \times \{CAPACITY(J)\} \times \{STYPES(J)\} \times \{SPEEDS(J)\} \times \{ULTIMES(J)\} \times \{LTIMES(J)\}) ,$$

$$P_{SHIP(J)} = ASSIGNMENTS \times CARGOS \times PORTS \times DISTANCE ,$$

$$F_{SHIP(J)} = \mathfrak{G}\{\mathfrak{C}_p : p \varepsilon P_{SHIP(J)})\} ,$$

$$M_{SHIP(J)} = \Gamma(\sigma_{SHIP(J)}) ,$$

$$T_{SHIP(J)} = I^{++} ,$$

$$\sigma_{SHIP(J)}(f,t)(x) = \sigma_{SHIP(J)}(c_{f(t-1)}, 1) \sigma_{SHIP(J)}(f,t-1)(x) \text{ if } t>0 ,$$
$$= x \text{ if } t=0 .$$

Having defined the system above the state transition function $\sigma_{SHIP(J)}$ must now be displayed in order to complete the definition. To do this, however, requires a look ahead at the structure of the port model. In particular the first component of the state set of the port model should be defined since it will be input to the ship system.

$$\Pi_1(\text{PORT}(K)(x)) \varepsilon \{\text{FULL,NOTFULL}\}, \text{ where } K \varepsilon I[1,\text{NPORT}] \ .$$

This is the status component and indicates whether or not a vessel may enter the port to use its docking facilities.

$$\Pi(\text{SACTION})\sigma_{\text{SHIP}(J)}(c_p,1)(x)$$

$= \Pi(\text{SACTION}(x))$ if $\Pi(\text{RELTIME}(J)(x)) > 0$.

$=$ SAILING if $\Pi(\text{SACTION}(J)(x) = \text{LOADING}$,

and $\Pi(\text{RELTIME}(J)(x)) = 0$,

and $\Pi_8(\text{CARGO}(J)(x)) = \text{READY}$,

where $\text{CARGO}(J) \varepsilon \text{CARGOS}$.

$=$ BALLAST if $\Pi(\text{SACTION}(J)(x)) = \text{UNLOADING}$,

and $\Pi(\text{RELTIME}(J)(x)) = 0$,

and $\Pi_1(\text{CARGO}(J)(x)) \neq (\text{OPORT}(J)(x))$,

where $\text{CARGO}(J) \varepsilon \text{CARGOS}$.

$=$ LOADING if $\Pi(\text{SACTION}(J)(x)) = \text{BALLAST}$,

and $\Pi(\text{RELTIME}(J)(x)) = 0$,

and $\Pi_1(\text{DPORT}(J)(x))(\text{PORTS}(p)) = \text{NOTFUL}$,

and $\Pi_8(\text{CARGO}(J)(x)) = \text{READY}$,

where $\text{CARGO}(J) \varepsilon \text{CARGOS}$.

$=$ LOADING if $\Pi(\text{SACTION}(J)(x)) = \text{UNLOADING}$,

and $\Pi(\text{RELTIME}(J)(x)) = 0$,

and $\Pi_1(\text{DPORT}(J)(x))(\text{PORTS}(p)) = \text{NOTFUL}$,

and $\Pi_8(\text{CARGO}(J)(x)) = \text{READY}$,

where $\text{CARGO}(J) \varepsilon \text{CARGOS}$.

$= \text{FREE if } \Pi(\text{SACTION}(J)(x)) = \text{FREE}$.

$(\text{RELTIME}(J))\sigma_{\text{SHIP}(J)}(c_p,1)(x)$

$= \Pi(\text{RELTIME}(J)(x)) - 1 \text{ if } \Pi(\text{RELTIME}(J)(x)) \neq 0$.

$= \Pi(\text{DISTANCE}(K,L)(p))/\Pi(\text{SPEEDS}(J,1)(x))$,

$\text{if } \Pi(\text{RELTIME}(J)(x) = 0$,

$\text{and } \Pi(\text{SACTION}(J)(x)) = \text{LOADING}$,

$\text{where } K = \Pi_1(\text{CARGO}(J)(x))$,

$\text{and } L = \Pi_2(\text{CARGO}(J)(x)) \quad \text{CARGO}(J) \varepsilon \text{CARGOS}$,

$= \Pi(\text{DISTANCE}(K,L)(p))/\Pi(\text{SPEEDS}(J,2)(x))$,

$\text{if } \Pi(\text{RELTIME}(J)(x)) = 0$,

$\text{and } \Pi(\text{SACTION}(J)(x)) = \text{UNLOADING}$,

$\text{where } K = \Pi(\text{OPORT}(J)(x))$,

$\text{and } L = \Pi_1(\text{CARGO}(J)(x))$,

$= \Pi_4(\text{CARGO}(J)(x))/\text{ULTIMES}(J)(x))$,

$\text{if } \Pi(\text{RELTIME}(J)(x)) = 0$,

$\text{and } \Pi(\text{SACTION}(J)(x)) = \text{SAILING}$,

$\text{and } \Pi_1(\text{DPORT}(J)(x))(\text{PORTS}(p) = \text{NOTFUL}$,

$\text{or } \Pi_1(\text{SACTION}(J)(x)) = \text{WAITING}$,

$\text{and } \Pi_1(\text{DPORT}(J)(x)(\text{PORTS}(p)) = \text{NOTFUL}$.

$= \Pi_4(\text{CARGO}(J)(x))/\text{LTIMES}(J)(x))$,

$\text{if } \Pi(\text{RELTIME}(J)(x)) = 0$,

$\text{and } \Pi(\text{SACTION}(J)(x)) = \text{BALLAST}$,

$\text{and } \Pi_8(\text{CARGO}(J)(x) = \text{READY}$,

$\text{and } \Pi_1(\text{DPORT}(J)(x))(\text{PORTS}(p)) = \text{NOTFUL}$,

$$\text{or } \Pi(\text{SACTION}(J)(x)) = \text{WAITING},$$
$$\text{and } \Pi(\text{CARGO}(J)(x)) = \text{READY},$$
$$\text{and } \Pi_1(\text{DPORT}(J)(x))(\text{PORTS}(p)) = \text{NOTFUL}.$$
$$= \Pi(\text{RELTIME}(J)(x)) + 1,$$
$$\text{if } \Pi(\text{SACTION}(J)(x) = \text{WAITING},$$
$$\text{and either } \Pi_8(\text{CARGO}(J)(x)) = \text{NOTREADY}$$
$$\text{or } \quad \Pi_1(\text{DPORT}(J)(x))(\text{PORTS}(p)) = \text{FULL}.$$

$$\Pi(\text{OPORT})\sigma_{\text{SHIP}(J)}(c_p,1)(x)$$
$$= \Pi(\text{DPORT}(J)(x)) \text{ if } \Pi(\text{SACTION}(J)(x)) = \text{BALLAST}$$
$$\text{or } \Pi(\text{SACTION}(J)(x)) = \text{SAILING},$$
$$\text{and } \Pi(\text{RELTIME}(J)(x)) = 0,$$
$$= \Pi(\text{DPORT}(J)(x)) \text{ otherwise}.$$

$$\Pi(\text{DPORT})\sigma_{\text{SHIP}(J)}(c_p,1)(x)$$
$$= \Pi_2(\text{CARGO}(J)(x)) \text{ if } \Pi(\text{SACTION}(J)(x)) = \text{BALLAST},$$
$$\text{and } \Pi(\text{RELTIME}(J)(x)) = 0.$$
$$= \Pi_1(\text{CARGO}(J)(x)) \text{ if } \Pi(\text{SACTION}(J)(x)) = \text{SAILING},$$
$$\text{and } \Pi(\text{RELTIME}(J)(x)) = 0.$$
$$= \Pi(\text{DPORT}(J)(x)) \text{ otherwise}.$$

$$\Pi(\text{CARGO}(J)\sigma_{\text{SHIP}(J)}(c_p,1)(x)$$
$$/= \text{CARGO}(J) \ni \text{CARGO}(I) \varepsilon \text{CARGOS}$$
$$\text{and } \Pi(\text{ASSIGNMENTS}(I,J)(p)) = \Pi(\text{ITRIP}(J)(x)),$$
$$\text{and } \Pi(\text{SACTION}(J)(x)) = \text{UNLOADING},$$
$$\text{and } \Pi(\text{RELTIME}(J)(x)) = 0.$$

$$\Pi(\text{ITRIP}(J)\sigma_{\text{SHIP}(J)}(c_p,1)(x)$$
$$= \Pi(\text{ITRIP}(J)(x)) + 1 \text{ if } \Pi(\text{SACTION}(J)(x)) = \text{UNLOADING} ,$$
$$\text{and } \Pi(\text{RELTIME}(J))(x) = 0 .$$
$$\Pi_L(J)\sigma_{\text{SHIP}(J)}(c_p,1)(x) = \Pi_L(x) \ \forall\ L = 7, 8, 9, 10, 11 .$$

The system $Z_{\text{SHIP}(J)}$ has now been completely defined and once again a check as to what assumptions were made in the above model is in order. The only assumption that was made was that a vessel would only be capable of traveling at two speeds: a loaded speed, and a ballast speed. This assumption was discussed earlier and it can be shown then if travel time is computed as follows,

SAILINGTIME = DISTANCE(OPORT,DPORT)/SPEEDS(J,1)

BALLASTTIME = DISTANCE(IPORT,OPORT)/SPEEDS(J,2) ,

that by adjusting the values of DISTANCE and SPEEDS the times computed for each vessel can be made to duplicate the times that are available in the records of most ship companies.

The system $Z_{\text{SHIP}(J)}$ will also have an output function $\zeta_{\text{SHIP}(J)}(x)$ defined on $S_{\text{SHIP}(J)}$ with values in $Q_{\text{SHIP}(J)}$. A schematic of the system appears in Figure 2.

The Port Model

The last basic entity of a maritime transportation network is a port. Let PORTS be the set as defined above but assume that

PORTS $\subset$ PSTATUS X PSHIP X ASHIP X DRAFT X MAXSHIPS X PTYPES,

where

PSTATUS = {FULL, NOTFUL}

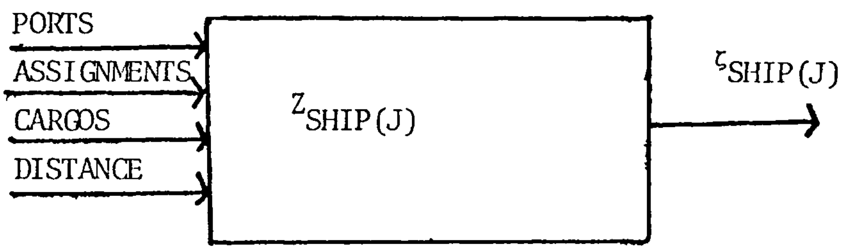

Figure 2. The ship system.

$$PSHIP = I\ [1, NSHIP]$$

$$ASHIP = I\ [1,NSHIP]$$

$$DRAFT = I^{++}$$

$$MAXSHIPS = I^{++} .$$

Then each $PORT(K) \varepsilon PORTS$ is of the form

$$PORT(k) = (PSTATUS(k), PSHIP(k), ASHIP(k), DRAFT(k), MAXSHIPS(k), PTYPES(k)) .$$

PSTATUS(k) is an element of the set PSTATUS and is the current status of the Kth port indicating whether or not the port may accept incoming vessels to load or unload cargo. ASHIP(k) is the number of ships that currently have the Kth port as a port of destination and is an element of the set ASHIP. PSHIP(k) is the number of ships that are currently using the Kth port's facilities to load or unload their cargo and is an element of the set PSHIP. DRAFT (k) is the maximum dead weight tonnage of ships that may enter the Kth port and is an element of the set DRAFT. MAXSHIPS(k) is the maximum number of vessels that may use the kth port's facilities at one time and is an element of the set MAXSHIPS. Let

$$Z_{PORT(K)} = \{S_{PORT(K)}, P_{PORT(K)}, F_{PORT(K)}, M_{PORT(K)}, T_{PORT(K)}, \sigma_{PORT(K)}\},$$

where

$$S_{PORT(K)} = (\{PSTATUS(K)\} X \{PSHIP(K)\} X \{ASHIP(K)\} X \{DRAFT(K)\} X \{MAXSHIPS(K)\}) ,$$

$$P_{PORT(K)} = CARGOS\ X\ SHIPS$$

$$F_{PORT(K)} = \mathcal{g}(\{c_p : p \varepsilon P_{PORT(K)}\}) \ ,$$

$$M_{PORT(K)} = \Gamma(\sigma_{PORT(K)}) \ ,$$

$$T_{PORT(K)} = I^{++} \ ,$$

$$\sigma_{PORT(K)}(f,t)(x) = \sigma_{PORT(K)}(c_{f(t-1)},1)$$

$$\sigma_{PORT(K)}(f,t-1)(x) \text{ if } t \neq 0 \ ,$$

$$= x \text{ if } t=0 \ .$$

Port State Transitions

$\Pi(\mathrm{PSTATUS}(K))\sigma_{PORT(K)}(c_p,1)(x)$

$= \mathrm{FULL}$ if $\Pi(\mathrm{PSHIP})(K)(x)) = \Pi(\mathrm{MAXSHIP}(K)(x))$,

$= \mathrm{NOTFUL}$ if $\Pi(\mathrm{PSHIP}(K)(x) < \Pi(\mathrm{MAXSHIP}(K)(x))$,

$= \Pi(\mathrm{PSTATUS}(K)(x))$ otherwise .

$\Pi(\mathrm{PSHIP}(K)(x))\sigma_{PORT(K)}(c_p,1)(x)$

$= \Pi(\mathrm{PSHIP}(K)(x)) + 1$ if $\Pi_1(\mathrm{SHIP}(J)(p)) = \mathrm{SAILING}$ or

BALLAST ,

and $\Pi_3(\mathrm{SHIP}(J)(p)) = 0$,

and $\Pi(\mathrm{PSTATUS})(K)(x)) = \mathrm{NOTFUL}$.

$= \Pi(\mathrm{PSHIP}(K))(x) - 1$ if $\Pi_1(\mathrm{SHIP}(J)(p)) = \mathrm{LOADING}$ or

UNLOADING ,

and $\Pi_3(\mathrm{SHIP}(J)(p)) = 0$,

where $\mathrm{SHIP}(J) \varepsilon \mathrm{SHIPS}$.

$= \Pi(\mathrm{PSHIP}(K))x)$ otherwise .

$\Pi(\mathrm{DRAFT}(K))\sigma_{PORT(K)}(c_p,1)(x) = \Pi(\mathrm{DRAFT}(K)(x))$.

$\Pi(\mathrm{MAXSHIPS}(K))\sigma(c_p,1)(x) = \Pi(\mathrm{MAXSHIPS}(K)(x))$.

$$\Pi(\text{ASHIP}(K))\sigma_{\text{PORT}(K)}(c_p,1)(x)$$
$$= \Pi(\text{ASHIP}(K)(x) + 1 \text{ if } \Pi_1(\text{SHIP}(J)(p)) = \text{LOADING}$$
$$\text{or UNLOADING ,}$$
$$\text{and } \Pi_3(\text{SHIP}(J)(p)) = 0 \text{ ,}$$
$$\text{and } \Pi_5(\text{SHIP}(J)(p)) = K \text{ .}$$
$$= \Pi(\text{ASHIP}(K)(x) - 1 \text{ if } \Pi_1(\text{SHIP}(J)(p) = \text{LOADING or}$$
$$\text{UNLOADING ,}$$
$$\text{and } \Pi_3(\text{SHIP}(J)(p)) = 0 \text{ ,}$$
$$\text{and } \Pi_5(\text{SHIP}(J)(p)) = K \text{ .}$$

The port model also has an output function $\zeta_{PORT(K)}(x)$ defined on $S_{PORT(K)}$ with values in a set $Q_{PORT(K)}$. Input to this system consists of the sets SHIPS and CARGOS. A schematic of $Z_{PORT(K)}$ appears in Figure 3.

Now that the three basic entities of the transportation system have been modeled a numerical example will serve to clarify the operation of the system. Let the total transportation system consist of 3 cargoes, 2 ships, and 2 ports where

$$\text{CARGOS} = \{\text{CARGO}(1), \text{CARGO}(2), \text{CARGO}(3)\} \text{ and ,}$$
$$\text{SHIPS} = \{\text{SHIP}(1), \text{SHIP}(2)\} \text{ and ,}$$
$$\text{PORTS} = \{\text{PORT}(1), \text{PORT}(2)\} \text{ .}$$

At time period DATE = 0, let the initial states of the system be as follows:

$$S_{\text{CARGO}(1)} = \{1 \text{ X } 2 \text{ X OIL X } 60000 \text{ X } 10 \text{ X } 20 \text{ X } 40 \text{ X NOTREADY X } 0 \text{ X } 0 \text{ X } 1\} \text{ ,}$$
$$S_{\text{CARGO}(2)} = \{2 \text{ X } 1 \text{ X OIL X } 100000 \text{ X }) \text{ X } 15 \text{ X } 30 \text{ X READY X } 0 \text{ X } 0 \text{ X } 0\} \text{ ,}$$

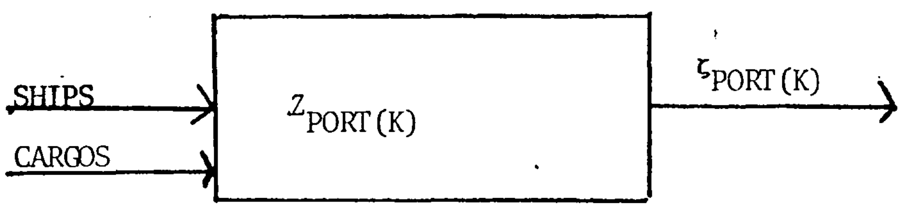

Figure 3. The port system.

$S_{CARGO(3)}$ = {1 X 2 X OIL X 70000 X 3 X 32 X 40 X NOTREADY X 0 X 0 X 0} ,

and

$S_{SHIP(1)}$ = {BALLAST X 1 X 3 X 2 X 1 X 1 X 70000 X OIL X 30,40 X {30000, 40000} X {30000, 40000}} ,

$S_{SHIP(2)}$ = {BALLAST X 2 X 2 X 1 X 2 X 1 X 100000 X OIL X 30,40 X {30000, 40000} X {30000, 40000}}

and

$S_{PORT(1)}$ = {NOTFUL X 0 X 1 X 75000 X 2 X OIL,COAL} ,

$S_{PORT(2)}$ = {NOTFUL X 0 X 1 X 120000 X 1 X OIL} .

Let the function ASSIGNMENTS have values as follows:

$$\text{ASSIGNMENTS}(I,J) = \begin{matrix} 1 & 0 \\ 0 & 2 \\ 0 & 1 \end{matrix}$$

Thus, the assignments indicate that cargo 1 has been assigned to ship 1, cargo 2 to ship 2, and cargo 3 to ship 2. Let the distance between ports 1 and 2 be 100 miles; then

DISTANCE(1,2) = 100.

The initial states indicate that cargo 1 is to be picked up at port 1 and delivered to port 2. The type of cargo is oil and there is 60000 tons to be moved. The cargo must be picked up between time periods 10 and 20 and delivered before time period 40, if a penalty is to be avoided. The status of the cargo at time period 0 is NOTREADY and it is to be picked up by ship 1. Cargo 2 is a 100000 ton parcel of oil to be picked up at port 2 and delivered to port 1. The cargo must be picked up between time periods 0 and 10 and delivered before time period 30. The status of the cargo at time period 0 is READY. Cargo 3 is a 70000 ton parcel of oil to be picked up at port 1 and delivered to port 2. The cargo must be picked up between time periods 3 and 32 and must be delivered before time period 40. The initial status of the cargo is NOTREADY. Ship 1 is initially traveling BALLAST between ports 2 and ports 1, there is 3 time units remaining in its voyage and it will pick up cargo 1 at port 1. Ship 1 is capable of carrying 70000 tons of oil, it can travel at 30 miles per hour loaded and 40 miles per hour ballast. A total of 30000 tons per time unit can be loaded onto vessel 1 at port 1, 40000 tons per time unit can be loaded at port 2, 30000 tons per time unit can be unloaded at port 1 and 40000 at port 2. Ship 2 is traveling ballast between ports 1 and 2 and there is 2 time units remaining in its trip. Ship 2 can carry a load of 100000 tons of oil with a loaded speed of 30 mph and a ballast speed of 40 mph. The loading and unloading rates at ports 1 and 2, respectively are 30000 tons per unit time, 40000 tpu, 30000 tpu and 40000 tpu. Port 1 is not full--it can accommodate 2 ships at one time--and currently 1 ship has port 1 as a destination. The port can service ships in the 75000-ton class and can load or unload cargoes of oil or coal. Port 2 is also not full--it can accommodate only 1 vessel at a time--and currently has 1 ship sailing to enter it. The port can service ships up to 120000 tons and can load or unload only oil cargoes. Only the changes in the states of ship 2, port 2 and cargo 2 will be described here.

During time periods 1, 2, and 3 the only change in state is in ship 2. During this time the RELTIME component decreases from 3 to 0.

In time period 4 the values of the dynamic components of the state of ship 2 have values as follows:

(LOADING X 1 X 3 X 2 X 1 X 1) .

The values of the components of the state of cargo 2 are:

(2 X 1 X OIL X 100000 X 0 X 15 X 30 X PICKEDUP X 4 X 0 X 2) .

The state of port 2 is:

(FULL X 1 X 1 X 120000 X 1 X OIL) .

At time period 7 ship 2 has completed loading and the states of ship 2, cargo 2, and port 2 are:

(SAILING X 2 X 3 X 2 X 1 X 1) ,

(2 X 1 X OIL X 100000 X 0 X 15 X 30 X PICKEDUP X 4 X 0 X 2) ,

(NOTFUL X 0 X 0 X 120000 X 1 X OIL) .

Ship 2 will next unload cargo 2 at port 1, load cargo 3 at port 1, and deliver it to port 2. After unloading cargo 3 at port 2 ship 1 will then become FREE, thus indicating that it has completed all of its assignments and is free to be scheduled again.

The Basic System

Now that each of the three basic entities of a maritime transportation network have been defined and modeled it should become clearer as to what contribution each of the subsystems must make in order to establish the total system. Their interaction can be obtained from inspection of the inputs to each subsystem and outputs from each subsystem. The general concept of this interaction is shown in Figure 4 but the interaction must also be defined mathematically in order to

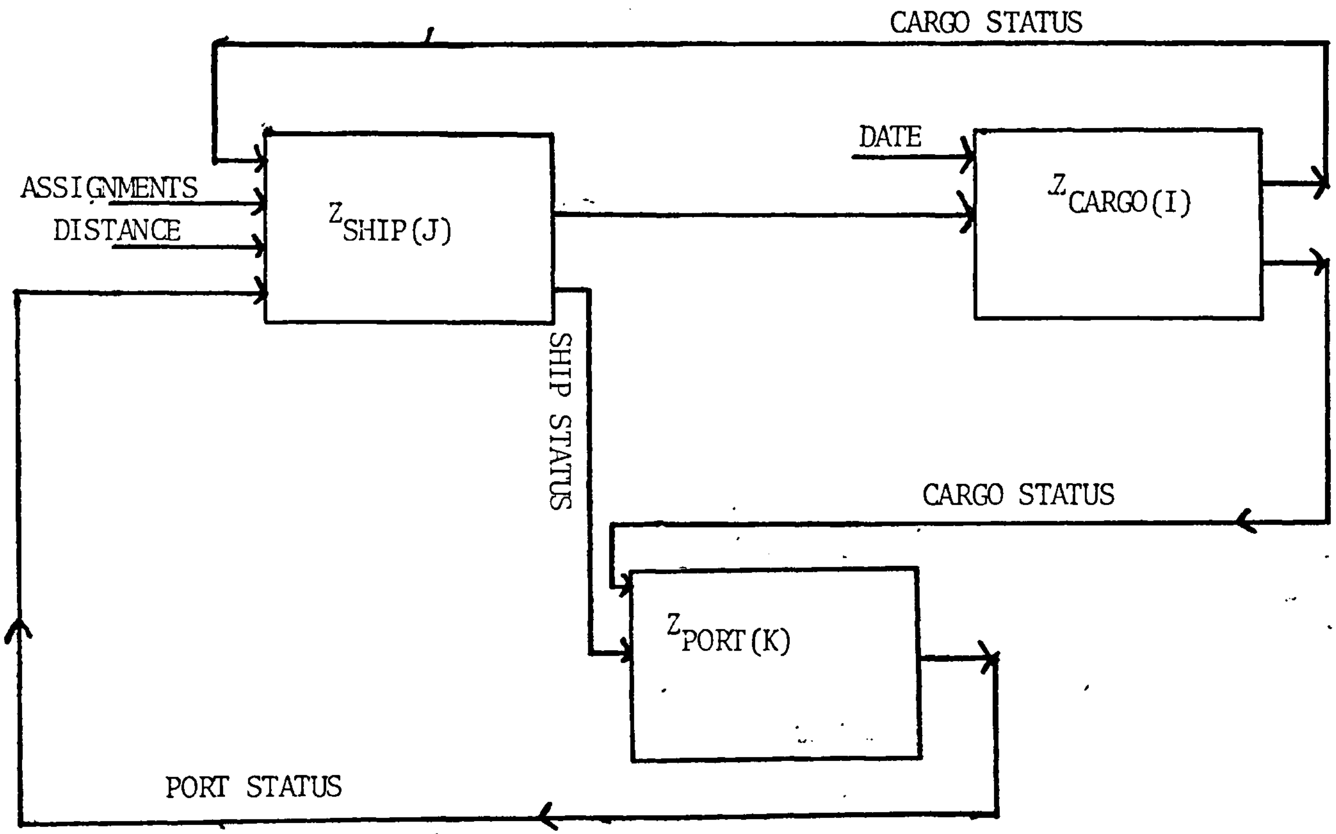

Figure 4. The basic system.

keep to the original premise of mathematical rigorousness. For this purpose, each of the subsystems input functions and output functions is defined mathematically.

Inputs to the Ship System

1. ASSIGNMENTS(J,I) $= \mathfrak{F}(I[1,NCARGO] \times I[1,NSHIP]), I^{++})$
2. $\zeta_{PORT(K),SHIP(J)}(x)$
3. $\zeta_{CARGO(J),SHIP(J)}(x)$
4. DISTANCE $= \mathfrak{F}(I[1,NPORT] \times I[1,NPORT], I^{++})$

Output from the Ship System

1. $\zeta_{SHIP(J),CARGO(I)}(x) = {}_1(SHIP(J)(x)) \times \Pi_2(SHIP(J)(x)) \times \Pi_3(SHIP(J)(x))$, if $\Pi_2(SHIP(J)(x)) = I$.
   $= 0$ otherwise .
2. $\zeta_{SHIP(J),PORT(K)}(x) = \Pi_1(SHIP(J)(x)) \times \Pi_3(SHIP(J)(x)) \times \Pi_5(SHIP(J)(x))$,
   if $\Pi_5(SHIP(J)(x)) = K$.
   $= 0$ otherwise .

Inputs to the Cargo System

1. $\zeta_{SHIP(J),CARGO(I)}(x)$, as defined above.
2. $\zeta_{DATE} = I^{++}$.

Output from the Cargo System

1. $\zeta_{CARGO(I),SHIP(J)}(x) = \Pi_2(CARGO(J)(x))$.
   $= 0$ otherwise.
2. $\zeta_{CARGO(I),PORT(K)}(x) = \Pi_8(CARGO(J)(x))$.

Input to the Port System

1. $\zeta_{SHIP(J),PORT(K)}(x)$, as defined above.
2. $\zeta_{CARGO(I),PORT(K)}(x))$, as defined above.

Output from the Port System

1. $\zeta_{PORT(K),SHIP(J)}(x) = \Pi_1(PORT(K)(x))$,
   if $\Pi_5(SHIP(J)(p)) = K$.
   $= 0$ otherwise.

The subsystem $Z_{SHIP(J)}$ has three inputs: a free input ASSIGNMENTS which establishes the Jth vessel's cargo assignments during the scheduling period, an input from the subsystem $Z_{CARGO(I)}$ which conveys the Ith cargo's status, and an input from the subsystem $Z_{PORT(K)}$ which conveys the status of the Kth port. This connection is true, however, only if the Ith vessel is beginning the trip on which it is to pick up the Jth cargo where the Kth port is the port of origin of the Jth cargo, or when the Ith vessel is traveling loaded with the Jth cargo, where the Kth port is the port of destination for cargo J. Otherwise there is no connection between SHIP(I), CARGO(J), and PORT(K). Thus a vessel obtains its current cargo assignment from the ASSIGNMENTS function and then the vessel need only know the status of the cargo it has been assigned and the status of the ports of origin and destination of this cargo in order to complete the assignment.

The subsystem $Z_{CARGO(I)}$ has a total of two inputs: a free input DATE which has a value equal to the total number of time units that the system has been running, and an input from the Jth vessel, if the Jth vessel has been assigned to pick up the Ith cargo. The input from $Z_{SHIP(J)}$ described above mathematically, represents the vessel's current action, the current cargo that the vessel is carrying, and the time units remaining until the vessel changes action.

The subsystem $Z_{PORT(K)}$ has two inputs: one input is from the ship subsystem, and represents the current action of the ship, the time units remaining until it changes action and the Jth vessel's current port of destination. The other input is from the cargo system and is the status of the Ith cargo.

Therefore, it appears that the subsystems $Z_{PORT(K)}$, and $Z_{CARGO(I)}$ have a more passive role in the total system than does the subsystem $Z_{SHIP(J)}$, as they merely monitor the vessel that is coming to the Kth port to pick up the Ith cargo, whereas the ship system is an active system driven by the ASSIGNMENTS input. It is this function ASSIGNMENTS that determines the action of a given vessel and therefore the action of the total system. Due to its importance a more detailed discussion of this function is in order.

Ideally this function must be dynamic with time. That is, the assignment of vessels to cargoes at time "t" must reflect the action of the system which has taken place between time "0" and "t-1," and furthermore must consider the action which will take place f m time "t+1" to the end of the scheduling period. If this function is not dynamic with time then at the start of the scheduling period each vessel is given a list of cargoes that it must pick up and deliver in a particular order.

These initial assignments would not change even if during the operation of the system it became apparent that these assignments were not optimal or even not feasible to carry out. Thus it appears that there must be another system involved, that is, one that is capable of changing the initial assignments at a given time, based on the current states of the vessels, ports, and cargoes. It is at this point that the crux of the scheduling problem becomes apparent. That is, the system that modifies the ASSIGNMENTS function during the operation of the system will be doing the optimization of the schedule, and therefore it must have some criterion for making the changes in assignments. In present practice this system is the human, who makes the assignments based on his judgment and past experience. It is also at this point where past attempts to automate the scheduling of vessels have failed, either by trying to duplicate human judgment and experience on a computer, or by calling upon some mathematical technique to try to make an optimal decision. In any case, it is at this point that the past solution techniques have become impractical for use in the real world. Therefore, it is now time to answer the question which was proposed at the beginning of this thesis: what is the best method of solving the scheduling problem on a computer? To try and duplicate human judgment and experience, to search for some optimization to manipulate the ASSIGNMENTS function and thus the total profit of a particular schedule, or to let the human express his judgments to the computer for evaluation?

In order to answer this question this author decided that some experience with the operation of the models described above was necessary. Therefore, the three subsystems $Z_{SHIP(J)}$, $Z_{PORT(K)}$, and $Z_{CARGO(I)}$, were implemented via FORTRAN IV computer programs so that some experimentation could be performed.

The results of these initial experiments showed that the operation of the three subsystems over a scheduling period of 400 days could be simulated in about .9 seconds of central processor time on the CDC 6400 at the University of Arizona. The first simulation consisted of a system of three vessels, five ports, and fifteen cargoes. In the second experiment the system consisted of ten vessels, five ports, and thirty cargoes; this simulation required about 1.9 seconds of central processor time on the 6400. In both of the above experiments the assignments of vessels to cargoes were decided upon initially, and read into memory at the start of the simulation. Thus the assignments were not dynamic with time, but rather the system carried out the actions dictated by the assignments at the start of the scheduling period, and thus merely evaluated one particular schedule. The speed of execution of the programs was impressive but it was clear that the computer was not doing enough by just evaluating one fixed schedule per run. It was decided, however, to take advantage of the speed of these programs in evaluating schedules and in addition to develop a system that would be capable of modifying the assignments as the system was operating. With the addition of this assignment system it would be easier to evaluate different schedules by making the assignment routine more flexible. Thus, the question has been answered. A best solution technique for the scheduling problem is one that has the following properties:

1. The system should be fast in execution and therefore low in cost to operate.
2. The system should take advantage of the high speed digital computer by making it evaluate a particular schedule.
3. The system should be easy to use by a human. This means that it should be easy for him to direct the system to evaluate a certain schedule that is based on his judgment.
4. The system should minimize the amount of work needed by the human in order to optimize a schedule, without taking over any of his judgmental work.

Keeping in mind the above four considerations the final form of the solution technique was developed. This meant that two more subsystems needed to be defined, modeled, and implemented: one subsystem to eliminate all non-feasible schedules from computer evaluation and thus from human consideration, and another subsystem to modify the ASSIGNMENTS function during the execution of the system. The definition of these two subsystems follows.

Elimination of Non-Feasible Schedules

Let Z_{POSSIB} be a system that enumerates all possible assignments for the Ith vessel, where

$$Z_{POSSIB} = (S_{POSSIB}, P_{POSSIB}, F_{POSSIB}, M_{POSSIB}, T_{POSSIB}, \sigma_{POSSIB}),$$

and

$$S_{POSSIB} = (\text{PASSIGN(J,I)}),$$

$$P_{POSSIB} = (\text{SPEEDS(I) X DISTANCE X PREST X CAPACITY(I)}$$

$X\ STYPES(I)\ X\ CARGO(J)\ X\ ASSIGNMENTS(J,I))\quad J\varepsilon I[1,NCARGO],$

and $\quad$ and $I\varepsilon I\ [1,NSHIP]$,

$$F_{POSSIB} = \mathcal{g}(\{c_p : p\varepsilon P_{POSSIB}\}) ,$$

$$M_{POSSIB} = \Gamma(\sigma_{POSSIB}) ,$$

$$T_{POSSIB} = I^{++} ,$$

$$\sigma_{POSSIB}(f,t)(x) = \sigma_{POSSIB}(c_{f(t-1)},1)\sigma_{POSSIB}(f_1 t-1)(x)$$

$$\text{if } t>0 .$$

$$= x \text{ if } t=0.$$

State Transitions

$\Pi(PASSIGN(J,I))\sigma(c_p,1)(x) = 1$,

if $\Pi_3(CARGO(J)(p))\varepsilon\Pi(STYPES(I)(p))$,

and $\Pi(ASSIGNMENTS(J,I)(p)) > 0$,

and $\Pi_4(CARGO(J)(p)) \leq \Pi(CAPACITY(I)(p))$,

and $\Pi(CAPACITY(I)(p)) \leq PREST(OPORT,DPORT)$,

where $OPORT = \Pi_1(CARGO(J)(p))$,

and $DPORT = \Pi_2(CARGO(J)(p))$,

where $PREST(OPORT,DPORT) = MIN(DRAFT(OPORT),$

$CANALS(OPORT,DPORT))$,

where $CANALS(OPORT,DPORT) = I^{++}$,

and $\Pi(DISTANCE(OPORT,DPORT)(p))/\Pi(SPEEDS(I,1)(p))$

$\leq (DUEDATE-LDATE)$,

where $DUEDATE = \Pi_7(CARGO(J)(p))$,

and $LDATE = \Pi_6(CARGO(J)(p))$.

$\Pi(PASSIGN(J,I))\sigma(c_p,1)(x) = 0$ otherwise.

The above system Z_{POSSIB} enumerates all possible assignments and eliminates all that are not feasible. That is, if the type of the Jth cargo is compatible with the cargo types that the Ith vessel can carry, and if the capacity of the Ith vessel is large enough so that it can carry the Jth cargo, and if the Ith vessel is not too large to enter the ports of origin and destination of the Jth cargo, and if the speed of the Ith vessel is sufficient to deliver the Jth cargo to its port of destination on time, then the assignment of cargo J to vessel K is considered feasible. If, however, a possible assignment fails any of the above tests then that assignment is considered non-feasible. The output function $\zeta_{POSSIB}(x)$ will now be described.

Let $\zeta_{POSSIB}(x) = \pi(\text{ASSIGNMENTS}(J,I)(p))$, if $\pi(\text{ASSIGNMENTS}(J,I)(p)) > 0$, and $\pi(\text{PASSIGN}(J,I)(x)) \neq 0$.

$= 0$ otherwise.

In words, let the above system's contribution to the total system be the ASSIGNMENTS function such that there are no non-feasible assignments enumerated by this function. Thus the output of the system is the initial values of the ASSIGNMENTS function as specified by a human except if the human has specified any non-feasible assignments. If non-feasible assignments were specified then they are eliminated by the system.

The heuristic meaning of the initial values of the ASSIGNMENTS function are particularly important and should be discussed now.

The function ASSIGNMENTS conveys the assignments to the vessels in the system as follows: let ASSIGNMENTS(J,I) = 0 if the Ith vessel is not to be considered by the system as a possible carrier of the Jth cargo. Thus ASSIGNMENTS(J,I) could be zero under the following conditions:

1. The initial input to Z_{POSSIB} was such that ASSIGNMENTS(J,I)=0.
2. The initial input to Z_{POSSIB} was such that ASSIGNMENTS(J,I)$\neq$0, but the assignment of the Ith vessel to carry the Jth cargo was not feasible.

Furthermore, let ASSIGNMENTS(J,I) = K, where $K \varepsilon I^{++}$ if the system is to consider the Ith vessel as a possible carrier of the Jth cargo for its Kth assignment. Thus ASSIGNMENTS(J,I) could be equal to K under one condition: if the initial input to Z_{POSSIB} was such that ASSIGNMENTS(J,I) = K, and the assignment of the Jth cargo to the Ith vessel was found to be feasible.

Thus the human inputs the set of assignments that he wants evaluated by the system. Z_{POSSIB} inspects the assignments for feasibility eliminating any assignments that are not feasible.

Modification of Initial Assignments

Let Z_{ASSMNT} be a system that modifies the assignments during the scheduling period, where

$$Z_{ASSMNT} = \{S_{ASSMNT}, P_{ASSMNT}, F_{ASSMNT}, M_{ASSMNT}, T_{ASSMNT}, \sigma_{ASSMNT}\},$$

where

$$S_{ASSMNT} = \text{ASSIGNMENTS(J,I) X SHIP(I))} ,$$

$$P_{ASSMNT} = \text{SHIPS} ,$$

$$F_{ASSMNT} = \mathbf{g}(\{c_p : p \varepsilon P_{ASSMNT}\}) ,$$

$M_{ASSMNT} = \Gamma(\sigma_{ASSMNT})$,

$T_{ASSMNT} = I^{++}$,

$\sigma_{ASSMNT}(f,t)(x) = \sigma_{ASSMNT}(c_{f(t-1)},1)(\sigma_{ASSMNT}(f,t-1)(x))$ if $t \neq 0$.

$= x$ if $t = 0$.

Assignment State Transitions

$\Pi(\text{ASSIGNMENTS}(J,I))\sigma_{ASSMNT}(c_p,1)(x)$

$= 0 \ \forall \ L \neq I \ni L \in I[1,\text{NSHIP}]$ and $I \in I\ [1,\text{NSHIP}]$,

if $\Pi(\text{ASSIGNMENTS}(J,I)(x)) = \Pi_6(\text{SHIP}(I)(x))$,

and $\Pi_1(\text{SHIP}(I)(x)) = \text{UNLOADING}$,

and $\Pi_3(\text{SHIP}(I)(x)) = 0$,

and $\Pi_2(\text{SHIP}(I)(x)) = J$,

$= -1$ for $L = I$ if $\Pi(\text{ASSIGNMENTS}(J,I)(x)) = \Pi_6(\text{SHIP}(I)(x)$

$= \Pi_6(\text{SHIP}(I)(x))$,

and $\Pi_1(\text{SHIP}(I)(x)) = \text{LOADING}$, $\Pi_2(\text{SHIP}(I)(x)) = J$,

and $\Pi_3(\text{SHIP}(I)(x)) = 0$.

$\Pi_6(\text{SHIP}(I))\sigma_{ASSMNT}(c_p,1)(x)$

$= \Pi_6(\text{SHIP}(I)(x)) + 1$ if $\Pi_6(\text{SHIP}(I)(x))$

$\neq \Pi(\text{ASSIGNMENTS}(J,I)(x))$,

and $\Pi_6(\text{SHIP}(I)(x)) < \text{NCARGO}$.

$= \Pi_6(\text{SHIP}(I)(p))$ otherwise .

The above system modifies the assignments during the scheduling period by modifying the values of the function ASSIGNMENTS. The modifications are described mathematically by the above system's state transitions function σ_{ASSMNT}. In words the system eliminates the Jth cargo from further consideration as a cargo for other vessels when the Ith vessel begins its voyage to pick up the Jth cargo. Also if cargo J has already been picked up by another vessel before the Ith vessel attempts to schedule it the system chooses another cargo for the Ith vessel to carry at that time. The modifications to the assignments may not seem to be very sophisticated but these modifications enable the system to evaluate a large number of alternative schedules in one short computer run. The modifications also are done in a manner that does not presuppose any human judgmental powers, human insight, or human experience. The modifications are done in a manner that can be described mathematically based on the original values of the ASSIGNMENTS function and the current state of the system at modification time. The human judgments, etc., that are needed in order to optimize a schedule are expressed by the human when he chooses the original values for the ASSIGNMENTS function. The output function ζ_{ASSMNT} may be defined as follows:

$$\zeta_{ASSMNT}(x) = x \ .$$

Thus this system contributes the updated assignment function to the total system.

Economic Evaluation of a Schedule

In order to complete the total system description there must be another subsystem that evaluates the worth of a particular schedule. The worth will be computed on an individual assignment basis, i.e., a profit figure will be computed for every assignment that a vessel carries out. The sum of this profit figure for all vessels and all assignments will be a total value assigned to a particular schedule. Let

$$\text{INCOME}(I,J) = \text{PRICE}(\text{ITYPE},\text{DPORT})*\text{AMOUNT} ,$$

where

$$\text{ASSIGNMENTS}(K,L) = J \text{ and } \text{ASSIGNMENTS}(K,M) = 0 \; M \neq L,$$
$$M \varepsilon I[1,\text{NSHIP}],\; K \varepsilon I[1,\text{NCARGO}] ;$$

then let

$$\begin{aligned}
&\text{BALLAST}(I,J) &&= I^{++}, \\
&\text{SAILING}(I,J) &&= I^{++}, \\
&\text{LOADING}(I,J) &&= I^{++}, \\
&\text{UNLOADING}(I,J) &&= I^{++}, \\
&\text{WAITING}(I,J) &&= I^{++}, \\
&\text{LATE}(I,J) &&= I^{++}, \\
&\text{SCOST}(I) &&= I^{++}, \\
&\text{LCOST}(I,L) &&= I^{++}, \text{ where } L=\Pi_1(\text{CARGO}(K)) , \\
&\text{UCOST}(I,L) &&= I^{++}, \text{ where } L=\Pi_2(\text{CARGO}(K)) , \\
&\text{PCOST}(K) &&= I^{++}, \\
&\text{WCOST}(I) &&= I^{++}, \\
&\text{ITYPE} &&= \Pi_3(\text{CARGO}(K)) \varepsilon \{\text{OIL},\text{COAL},\text{ORE}\} ; \\
&\text{AMOUNT} &&= \Pi_4(\text{CARGO}(K))=I^{++}, \\
&\text{OPORT} &&= \Pi_1(\text{CARGO}(K)) , \\
&\text{DPORT} &&= \Pi_2(\text{CARGO}(K)) , \\
&\text{NTRIP}(I) &&= I^{++}, \\
&\text{PRICE}(\text{ITYPE},\text{DPORT}) &&= \mathcal{F}([1,\text{NTYPES}] \times I[1,\text{NPORT}],\; I^{++}) .
\end{aligned}$$

Let BALLAST(I,J), SAILING(I,J), LOADING(I,J), UNLOADING(I,J), WAITING(I,J), be defined mathematically as above and let them represent the number of time units that vessel I spends with the action of BALLAST, SAILING, LOADING, UNLOADING, and WAITING respectively on its Jth assignment. Let LATE(I,J) be the time units that the Kth cargo was late when delivered by the Ith vessel on its Jth assignment. Let BCOST(I), SCOST(I), UCOST(I,L), LCOST(I,L), WCOST(I) be the cost per unit time associated with the actions of the Ith vessel. Let PCOST(K) be the penalty cost per unit time of the Kth cargo being delivered to its port of destination late. Let NTRIP(I) be the total number of assignments completed by the Ith vessel during the scheduling period, and let PRICE(ITYPE,DPORT) be the price per unit of cargo, of type ITYPE, received as income when shipped to port DPORT.

With the above definitions the worth of a particular assignment can now be computed. Let

PROFIT(I,J) = INCOME(I,J) - COST(I,J), where

INCOME(I,J) = PRICE(ITYPE,DPORT)*AMOUNT, and

COST(I,J) = SAILING(I,J)*SCOST(I) + BALLAST(I,J)*BCOST(I)
+ LOADING(I,J)*LCOST(I,OPORT) + UNLOADING(I,J)
*UCOST(I,DPORT) + WAITING(I,J)*WCOST(I)
+ LATE(I,J)*PCOST(K) .

Therefore PROFIT(I,J) is the total profit incurred by the Ith vessel on its Jth assignment. Hence the total profit over the entire scheduling period can be computed as follows:

$$\text{TOTALPROFIT} = \sum_{I=1}^{\text{NSHIP}} \sum_{J=1}^{\text{NTRIP(I)}} \text{PROFIT(I,J)} \; .$$

Let Z_{ECON} be a system that evaluates the worth of each assignment and hence the worth of an entire schedule. Let

$$Z_{ECON} = \{S_{ECON}, P_{ECON}, F_{ECON}, M_{ECON}, T_{ECON}, \sigma_{ECON}\},$$

where

$$S_{ECON} = (\text{BALLAST(I,J) X SAILING(I,J) X LOADING(I,J)}$$
$$\text{X UNLOADING(I,J) X WAITING(I,J) X LATE(I,J) X}$$
$$\text{X PROFIT(I,J))} \;,$$
$$P_{ECON} = (\text{SCOST(I) X BCOST(I) X LCOST(I,L) X UCOST(I,L)}$$
$$\text{X WCOST(I) X PCOST(L) X PRICE(ITYPE,DPORT) X SHIPS}$$
$$\text{X CARGOS)} \;,$$
$$F_{ECON} = \mathcal{P}\{c_p : p \; P_{ECON}\}) \;,$$
$$M_{ECON} = \Gamma(\sigma_{ECON}) \;,$$
$$T_{ECON} = I^{++} \;,$$
$$\sigma_{ECON}(f,t)(x) = \sigma_{ECON}(c_{p(t-1)},1)\,\sigma_{ECON}(f,t-1)(x) \text{ if } t \neq 0 \; .$$
$$= x \text{ if } t = 0.$$

Economic System State Transitions

$$\Pi(\text{BALLAST(I,J)})\,\sigma_{ECON}(c_p,1)(x)$$
$$= \Pi(\text{BALLAST(I,J)(x)}) + 1 \text{ if } \Pi_1(\text{SHIP(I)(p)}) = \text{BALLAST}$$
$$\text{and } \Pi_6(\text{SHIP(I)(p)}) = J \; .$$
$$= \Pi(\text{BALLAST(I,J)(x)}) \text{ otherwise} \; .$$
$$\Pi(\text{SAILING(I,J)})\,\sigma_{ECON}(c_p,1)(x)$$
$$= \Pi(\text{SAILING(I,J)(x)}) + 1 \text{ if } \Pi_1(\text{SHIP(I)(p)}) = \text{SAILING}$$
$$\text{and } \Pi_6(\text{SHIP(I)} = J \; .$$

$= \Pi(\text{SAILING }(I,J)(x))$ otherwise .

$\Pi(\text{LOADING}(I,J))\sigma_{ECON}(c_p,1)(x)$

$= \Pi(\text{LOADING}(I,J)(x)) + 1$ if $\Pi_1(\text{SHIP}(I)(p)) = \text{LOADING}$

and and $\Pi_6(\text{SHIP}(I)(p) = J$,

$= \Pi(\text{LOADING}(I,J)(x))$ otherwise .

$\Pi(\text{UNLOADING}(I,J))\sigma_{ECON}(c_p,1)(x)$

$= \Pi(\text{UNLOADING}(I,J)(x)) + 1$,

if $\Pi_1(\text{SHIP}(I)(x)) = \text{UNLOADING}$

and $\Pi_6(\text{SHIP}(I)(p)) = J$.

$= \Pi(\text{UNLOADING}(I,J)(x))$ otherwise .

$\Pi(\text{WAITING}(I,J))\sigma_{ECON}(c_p,1)(x)$

$= \Pi(\text{WAITING}(I,J)(x)) + 1$ if $\Pi_1(\text{SHIP}(I)(p)) = \text{WAITING}$,

$= \Pi(\text{WAITING}(I,J)(x))$ otherwise ,

and $\Pi_6(\text{SHIP}(I)(p)) = J$.

$\Pi(\text{LATE}(I,J))\sigma_{ECON}(c_p,1(x)$

$= \Pi_{10}(\text{CARGO}(L)(p)) - \Pi_7(\text{CARGO}(L)(p))$,

if $\Pi_1(\text{SHIP}(I)(p)) = \text{UNLOADING}$,

and $\Pi_6(\text{SHIP}(I)(p)) = J$,

and $\Pi_3(\text{SHIP}(I)(p)) = 0$,

and if $\Pi_{10}(\text{CARGO}(L)(p)) > \Pi_{11}(\text{CARGO}(L)(p)$.

$= 0$ otherwise .

$\Pi(\text{PROFIT}(I,J))\sigma_{ECON}(c_p,1)(x)$

$= \Pi(\text{PRICE}(\text{ITYPE},\text{DPORT}) * \Pi_4(\text{CARGO}(L)(p)) - \Pi_1(x)$

$* \Pi_1(p) - \Pi_2(x) * \Pi_2(p) - \Pi_3(x) * \Pi_3(p) - \Pi_4(x)$

$* \Pi_4(p) - \Pi_5(x) * \Pi_5(p) - \Pi_6(x) * \Pi_6(p)$,

$$\text{if } \Pi_1(\text{SHIP})(\text{I})(\text{p})) = \text{UNLOADING} ,$$
$$\text{and } \Pi_3(\text{SHIP}(\text{I})(\text{p})) = 0 ,$$
$$\text{and } \Pi_6(\text{SHIP}(\text{I})(\text{p})) = \text{J} ,$$
$$\text{where } \text{ITYPE} = \Pi_4(\text{CARGO}(\text{L})(\text{p})) ,$$
$$\text{and } \text{DPORT} = \Pi_2(\text{CARGO}(\text{L})(\text{p})) .$$
$$= 0 \text{ otherwise} .$$

The output from Z_{ECON} will be the profit incurred from vessel I on its Jth assignment and at the end of the scheduling peirod it will be the total profit from all vessels on all assignments. Mathematically, let

$$\zeta_{ECON}(x) = \Pi(\text{PROFIT}(\text{I},\text{J})(x)) ,$$
$$\text{if } \Pi_1(\text{SHIP}(\text{I})(\text{p}) = \text{UNLOADING} ,$$
$$\text{and } \Pi_3(\text{SHIP}(\text{I})(\text{p})) = 0 ,$$
$$\text{and } \Pi_6(\text{SHIP}(\text{I})(\text{p})) = \text{J} .$$
$$= \sum_{\text{I}=1}^{\text{NSHIP}} \sum_{\text{J}=1}^{\text{NTRIP(I)}} \Pi(\text{PROFIT}(\text{I},\text{J})(x)) ,$$
$$\text{if } \Pi_1(\text{SHIP}(\text{I})) = \text{FREE} \ \forall \ \text{I} \in \text{I}[1,\text{NSHIP}] .$$

The Total System

The total maritime transportation system has now been defined and modeled as a series of subsystems $Z_{SHIP(I)}$, $Z_{PORT(K)}$, $Z_{CARGO(J)}$, Z_{POSSIB}, Z_{ASSMNT}, Z_{ECON}. The solution technique has been chosen and is one that uses the speed of the digital computer to its fullest but does not require the computer to make judgmental decisions. The judgmental decisions that are vital to the proposed solution technique are left to the human who is using the system. However, unlike in the past, a well designed interface between the human and the system makes it possible for him to easily express his judgments for computer evaluation. It is in this manner that an optimal solution to the transportation scheduling problem may be found. The structure of the total system appears in Figure 5. The system thus has three free inputs:

1. ASSIGNMENTS input to Z_{POSSIB},
2. DATE input to $Z_{CARGO(I)}$, and
3. COST input to Z_{ECON}.

All other inputs to subsystems are output from another subsystem. A list of all system outputs follows:

1. $\zeta_{POSSIB,ASSMNT}$
2. $\zeta_{ASSMNT,SHIP(J)}$
3. $\zeta_{SHIP(J),CARGO(I)}$
4. $\zeta_{SHIP(J),PORT(K)}$
5. $\zeta_{SHIP(J),ASSMNT}$
6. $\zeta_{SHIP(J),ECON}$
7. $\zeta_{CARGO(I),SHIP(J)}$
8. $\zeta_{CARGO(I),ECON}$
9. $\zeta_{PORT(K),SHIP(J)}$
10. ζ_{ECON}

In the above list the first subscript indicates the system that the output came from while the second subscript indicates the system that will use it as an input. As an example: $\zeta_{PORT(K),SHIP(J)}$ is the output from $Z_{PORT(K)}$ and is input to $Z_{SHIP(J)}$. With the above summary

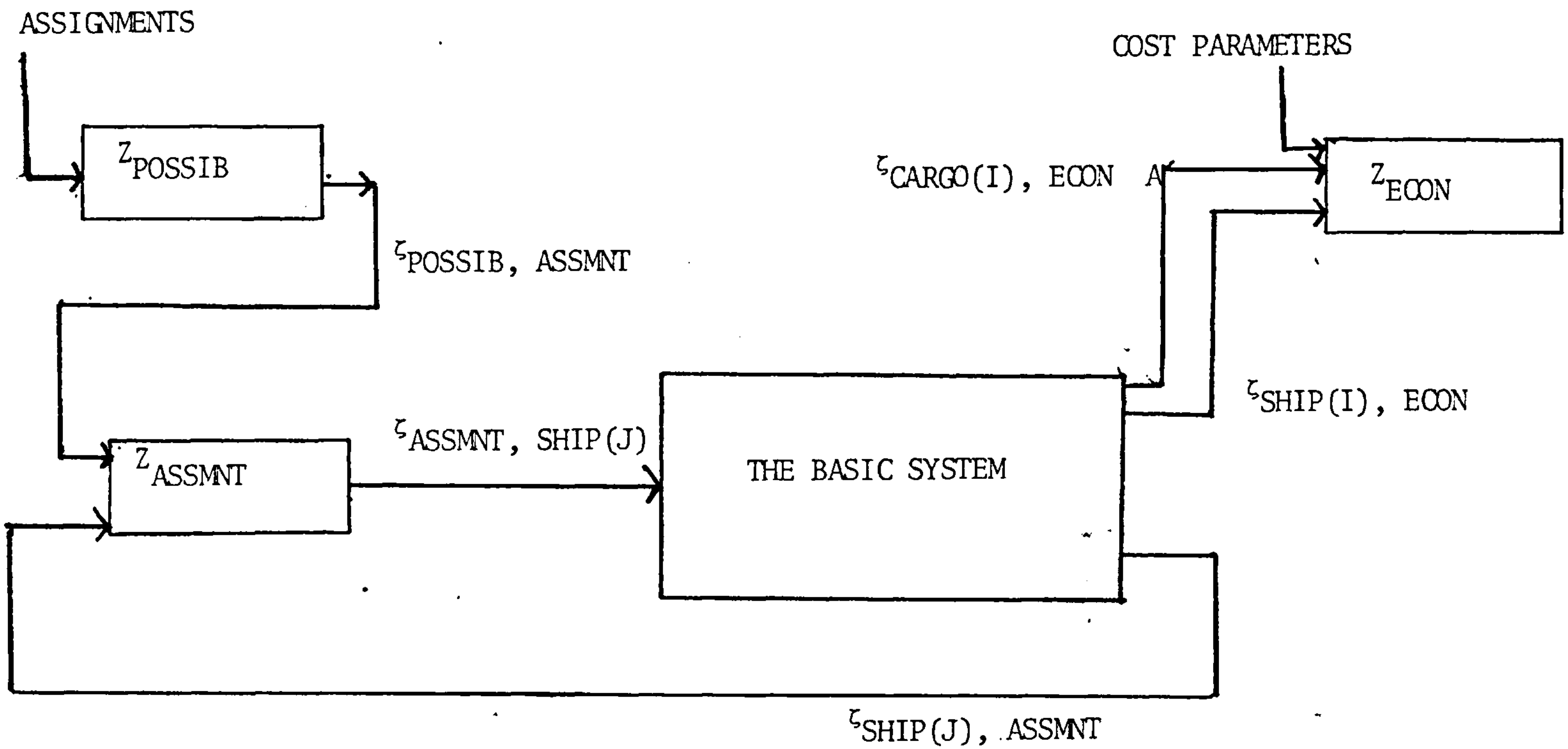

Figure 5. The total system.

the total system description is now complete and, as stated as a premise in this thesis, is mathematically rigorous but yet realistic.

CHAPTER 3

IMPLEMENTATION AND RESULTS

Having defined and modeled the total system the computer version of the system must now be developed, implemented, and experiments performed with it, so that its worth as a system for solving the transportation scheduling problem may be evaluated.

Programming

As a result of modeling the system using general system theory as the tool the actual task of implementing the system is reduced to one of programming each of the above six subsystems and their output functions. It was found that the coding of the systems into a computer language was quite simple as all that needed to be done was to describe the state transitions of each system in the programming language. The language chosen was a basic version of FORTRAN IV so as to make the programs that represent the system as machine independent as possible. Each of the six subsystems was programmed as a separate subroutine subprogram while the main program consisted of simple logic that controlled the order of execution of the system. One more subroutine was included, and its job was to initialize the system by reading in the initial states of the subsystems as well as other system parameters.

Memory Requirements

It is important in the development of any computerized system that the memory needed by the system does not make the system unusable. For this system if indeed the solution technique proves to be a good one but the storage requirements of the system are too large, the solution technique becomes unacceptable for use.

When defining memory requirements it is important to distinguish between the actual program requirements and the computer's system overhead. Thus

TOTALMEMORY = PROGRAM CODE + COMMON PARAMETERS + SYSTEM OVERHEAD ,

where

SYSTEM OVERHEAD = INPUT/OUTPUT BUFFERS + DATA FILE BUFFERS + SYSTEM'S PROGRAMS .

For the system developed here the memory requirements on the CDC 6400 are as follows:

I.	COMPUTER SYSTEM'S OVERHEAD		
	1. I/O BUFFERS	$4{,}000_8$	words
	2. DATA FILE BUFFER	$2{,}000_8$	words
	3. SYSTEM'S PROGRAMS	$5{,}000_8$	words
		$13{,}000_8$	words
II.	PROGRAM CODE	$6{,}400_8$	words

TOTAL REQUIREMENTS LESS COMMON = $21{,}400_8$ WORDS

The number of words required for the common parameters is a function of the number of ships, ports, and cargoes in the system. Hence mathematically then

$$\text{COMMON PARAMETERS} \quad \mathcal{F}(\text{NSHIP X NPORT X NCARGO, } I^{++}) ,$$

where this function can be approximated by the following function:

$$\text{F(NSHIP,NPORT,NCARGO)} \approx 12{*}\text{CARGO} + 37{*}\text{NPORT} + 2{*}\text{NPORT}^2 + 2{*}\text{NCARGO}{*}\text{NSHIP}) + 300_8 .$$

Thus the total memory requirements of the system can be described as follows:

$$\text{TOTALMEMORY} = 21{,}400_8 + \text{F(NSHIP,NPORT,NCARGO)}_8 .$$

A summary of the total memory requirements for various systems configurations is displayed in Table 1. As can be seen even for a very large configuration such as thirty ships, twenty ports, and eighty cargoes the total requirement on the CDC 6400 is only $37{,}500_8$ words, well below the total available to the user. The solution technique developed here, that relies on this system of computer programs will not be unacceptable because it uses too much memory, as its requirements are very minimal. The next test for the system to pass is one of cost to operate. That is, are the execution times of various system configurations low enough such that the cost of finding an optimal solution is acceptable to the industry.

Execution Time

Several experimental runs of the proposed system were conducted again using the CDC 6400 at The University of Arizona. These runs were

Table 1. Total memory requirements

NSHIP	NPORT	NCARGO	COMMON REQUIREMENTS	TOTAL
3	5	15	$1,400_8$	$23,000_8$
3	5	40	$2,300_8$	$23,700_8$
3	20	40	$3,500_8$	$25,400_8$
10	20	40	$5,100_8$	$26,500_8$
10	20	80	$13,400_8$	$35,000_8$
30	20	80	$16,100_8$	$37,500_8$

made only for the purpose of determining the amount of computer time that would be consumed by the various systems configurations. The times and cost shown in Table 2 are for one run of the system for each of the shown configurations. No attempt to optimize a schedule was made at this time. The total cost of finding an optimal solution for any of the shown configurations, however, will be the cost of one run times the number of runs needed to find the answer that is wanted.

As can be seen from the results in Table 2, the above experiments are very encouraging insomuch as the cost per run of the system was very small. The questions as to the cost of finding an optimal solution, as state above, is now reduced to one of how many computer runs will be needed for the human to obtain the information that he wants. Even if it took 50 computer runs this would incur a maximum computer cost of $136.00 where the possible savings to the user of the system could be in the range of millions of dollars. Therefore the very low ratio of computer cost to possible savings, the system should be extremely attractive to the shipping industry. This is true providing that the interface between the computer system of programs, and the human user is one that is acceptable to the human.

Use of the System

The computer programs that make up the above described system were designed with the human user in mind. The first phase is to set up the initialization data, such as initial states of all vessels, ports, and cargoes in the system. The next step is to set up the time and cost

Table 2. Execution times and costs

NSHIP	NPORT	NCARGO	DAYS SIMULATED	CP SECONDS	COST
3	5	15	200	2.86	.597
3	5	15	300	2.98	.596
3	20	40	500	4.60	.920
10	20	40	300	5.65	1.130
10	20	80	300	10.86	2.173
30	20	80	200	12.75	2.550
30	20	80	300	13.10	2.620

data for each vessel, etc. Once this initialization process is done and the data transferred to computer cards in the proper format then the system is ready to run. For the three ship, five port, and fifteen cargo configuration this initialization data takes about twenty-five data cards. Once this process is done, however, it need not be done again unless of course different time and cost data is to be used or different initial states are to be assigned to the vessels, ports, or cargoes. The next step is for the user to express his complete schedule for the system to evaluate. This is done by entering values onto one data card for each cargo. These values are actually the initial values of the ASSIGNMENTS function. The user need not be concerned about making non-feasible assignments as these will be eliminated by the system and thus not evaluated. Example assignments cards for a system with five cargoes and three vessels appear below.

CARD 1	0	0	1
CARD 2	1	0	0
CARD 3	0	1	0
CARD 4	2	3	2
CARD 5	2	2	3

The above five assignment cards are interpreted by the system as follows: Cargo 1 will be carried by vessel 3 if this assignment is feasible. Cargo 2 will be carried by vessel 1 if this assignment is feasible. Cargo 3 will be carried by vessel 2 if it is a feasible assignment. If any of the above assignments are not feasible then the cargo would not be picked up at all during the operation of the system. Cargo 4 is the second assignment for vessel 1, the third assignment for vessel 2, and the second assignment for vessel 3. Cargo 5 is the second assignment for vessel 1, the second assignment for vessel 2, and the third assignment for vessel 3. Thus competition between vessels for cargoes can be introduced into the schedule by assigning the same cargo to different vessels. In this version of the opeating system the vessel that is ready for the cargo the earliest is the vessel that actually will carry the cargo. Competition between cargoes for vessels can also be introduced into the schedule by assigning the same assignment number to two or more different cargoes for the same vessel. In the above example ships 1 and 3 have to compete for cargo 4, ships 1 and 2 also compete for cargo 5 and cargo 4 competes with cargo 5 for vessel one's second assignment. The competition between cargoes for vessels is resolved by assigning the cargo with lowest numerical value to the vessel.

The particular means by which the competition in a schedule is resolved was chosen arbitrarily so as not to violate the original premise that too many assumptions obscure the actual problem. The competition could also have been resolved by the system based on other criteria such as:

1. minimize ballast time, and/or

2. minimize the distance traveled, i.e., (loaded and ballast).

There are many others, but the decision as to which criteria is best for a particular schedule must be left up to human judgment and experience. With the above system this can be done by changing the numerical order of the cargoes and assignment number.

After a schedule was evaluated the user would look at the output and decide whether or not he was satisfied with the schedule. The criterion that he uses to decide if he is satisfied or not is not part of the computerized system, but rather the computerized system gives him enough information so that he can make his decision. If he is satisfied then he is done. If he is not satisfied then he may change the schedule in the manner he wishes by simply changing the values that are initially in the ASSIGNMENTS function. This involves only changing the five assignment cards and nothing else. The output that the user obtains from the system is not only the economic evaluation of the executed schedule but he is also able to examine the initial states of all vessels, ports, and cargoes and the present states of all vessels, ports, and cargoes every time a vessel completes an assignment. The final states of all vessels, ports and cargoes are also displayed at completion of the specified period of operation, as well as total time units each vessel spent in each action. Sample output from a three ship, five port, and 15 cargo configuration is shown in Appendix A. This author believes that enough information is displayed and done so in a manner that provides the human with enough information necessary to optimize a schedule.

Optimization of a Schedule

In order to show the power of the solution technique developed above it was decided to apply the technique to a specific problem and to obtain an optimal schedule.

The network has 3 vessels, 5 ports and 15 cargoes. The data used is completely fictitious but the values resemb real world data. Before beginning to optimize, it must be decided what criteria will be used to judge the value of a particular schedule. The following criteria will be the basis for this particular optimization test:

1. deliver all the cargoes using only the 3 initial vessels,
2. minimize total travel time, and
3. maximize profit.

The implications of the above criteria are that a ship will be allowed to deliver a cargo late but will have to pay the late charge provided for by the contract. Criteria 2 and 3 imply that, since all cargo parcels must be delivered, then the total benefit derived for any schedule is fixed; thus the problem becomes one of minimizing the cost.

The scheduling period will begin at time period "0" with vessel 1 assigned to cargo 1, vessel 2 assigned to cargo 2, and vessel 3 assigned to cargo 3. Thus, the initial state of the total system is fixed. The initial states of the ships, ports, and cargoes are displayed in Tables 3, 4, and 5. All possible or feasible assignments are shown in Table 6. These were computed by the programs, specifically the program that represents Z_{POSSIB}.

A total of five computer runs were made during the optimization of this specific problem. The five schedules that were evaluated are shown in Appendix B while, as noted above, all of the computer output from schedule 4 is shown in Appendix A.

Table 3. Ship initial states

ISHIP	ACTION	CARGO	RELTIME	OPORT	DPORT
1	SAILING	1	20	1	3
2	SAILING	2	10	5	2
3	BALLAST	3	5	1	3

Table 4. Port initial states

IPORT	STATUS	IN	COMING	SNUMB	CNUMB
1	NOTFUL	-0	-0	-0	-0
2	NOTFUL	-0	1	-0	-0
3	NOTFUL	-0	2	-0	-0
4	NOTFUL	-0	-0	-0	-0
5	NOTFUL	-0	-0	-0	-0

Table 5. Cargo initial states

CARGO	OPORT	DPORT	TYPE	AMOUNT	EDATE	LDATE	DUEDATE	STATUS	SHIP	PDATE	DDATE
1	1	3	OIL	50,000	0	5	40	PICKUP	1	2	0
2	5	2	OIL	60,000	0	6	50	PICKUP	2	5	-0
3	3	1	ORE	60,000	5	40	70	NREADY	-0	-0	-0
4	4	3	ORE	60,000	10	44	70	NREADY	-0	-0	-0
5	4	2	ORE	60,000	50	74	80	NREADY	-0	-0	-0
6	5	4	ORE	60,000	5	50	80	NREADY	-0	-0	-0
7	4	3	ORE	60,000	12	44	70	NREADY	-0	-0	-0
8	2	4	ORE	62,000	5	10	40	NREADY	-0	-0	-0
9	3	1	ORE	61,000	5	40	70	NREADY	-0	-0	-0
10	1	4	OIL	60,000	42	180	170	NREADY	-0	-0	-0
11	1	5	OIL	60,000	60	140	170	NREADY	-0	-0	-0
12	2	4	OIL	61,000	120	140	170	NREADY	-0	-0	-0
13	3	1	ORE	61,000	100	140	170	NREADY	-0	-0	-0
14	4	2	OIL	61,000	100	140	170	NREADY	-0	-0	-0
15	5	3	ORE	60,000	110	150	180	NREADY	-0	-0	-0

Table 6. Possible assignments

CARGO	SHIPS		
1	1	2	3
2	1	1	1
3	1	1	1
4	1	1	1
5	1	1	1
6	1	1	1
7	1	1	1
8	0	0	1
9	0	0	1
10	1	1	1
11	1	1	1
12	0	0	1
13	0	0	1
14	0	0	1
15	1	1	1

The results of the computer runs are summarized in Table 7. It can be seen that schedule 2 is the best in terms of the initial criterion set forth above. Schedule 1 was the first guess at an optimal solution; every other schedule was based on results of the previous computer run. Table 7 also shows that the maximum profit was obtained on the second run and the profit from run number 4 was the lowest of all the schedules.

These results reflect that the changes made to schedule 1 in order to arrive at schedule 2 resulted in the best schedule. Thus, the amount of information obtained from the computerized system in run number 1 enabled the user to make good decisions and testifies to the value of the interface between the human user and the program. Schedule 3 was an attempt to reduce the number of cargoes delivered late. This objective was obtained but ballast time increased and thus total cost. In schedule 4 (run number 4) drastic changes were made and resulted in a low profit. Run number 5 improved upon the previous run but it could be seen that the changes that needed to be made to schedule 5 would yield a schedule similar to schedule 2. Thus, the decision that 5 evaluations on this specific problem was enough and that schedule 2 might very well be an "optimal" schedule. This of course cannot be proved mathematically but an individual with much scheduling experience is likely to be able to choose what he considers to be the best schedule.

Table 7. Summary of optimization runs

SCHEDULE	DAYS SAILING	DAYS BALLAST	DAYS LOADING	DAYS UNLOADING	DAYS WAITING	COST	BENEFITS	PROFIT
1	272	230	49	23	0	16,242,600	45,460,000	29,217,400
2	268	191	48	23	0	15,333,400	45,460,000	30,126,600
3	262	204	50	23	0	15,435,500	45,460,000	30,024,500
4	272	233	50	23	0	16,359,600	45,460,000	29,100,400
5	272	213	48	23	0	15,840,600	45,460,000	29,619,400

CHAPTER 4

CONCLUSIONS

The major objective of this thesis was to analyze a maritime transportation network using a strict system theoretic approach. Using such a rigorous approach, it was hoped, would yield valuable formation as to what would be the best solution technique to the time constrained scheduling problem. The network was decomposed into several basic components, which were then modeled using general system theory as a tool for modeling. The models were developed realistically with a minimum of assumptions so as not to obscure the original problem. The models were then implemented via FORTRAN IV computer programs and it was concluded that due to the speed of exeuction of these programs that the best solution technique would be one that:

1. relied on the high-speed digital computer for evaluation of potential schedules,
2. relied on the human to make the critical decisions and to express his judgments using his past experience, and
3. provided a well designed interface between the human and the computer programs so that his judgment etc. could be easily expressed for computer evaluation.

It was then shown that the solution technique developed did satisfy the above three criteria and was also low in cost to operate.

The author realized, however, that the configuration of that system was very small, insomuch as only 3 vessels, 5 ports, and 15 cargoes were considered. A larger configuration, however, would merely mean more information would be obtained from the system and thus it would take more human time to enumerate schedules to be evaluated. It is important to note, however, that a larger system configuration would not make it more difficult for the human to express his judgments to the computer for evaluation. Thus, the size of the systems configuration does not affect the interface between the human and the computer.

Thus, the solution technique developed in this thesis is one which should be acceptable to the industry for which it was designed, that is, it is a technique that:

1. is low in cost to operate,
2. yields much valuable information,
3. needs interaction with a human, and
4. leaves all assumptions that need be made and all judgmental decisions to the human.

It is concluded, therefore, that this type solution technique is a desirable way to approach the solution of the transportation problem and that general system theory is a good tool for analysis and for modeling real world systems.

APPENDIX A

COMPUTER OUTPUT

INITIAL STATES

SHIPS

ISHIP	ACTION	CARGO	RELTIME	OPORT	DPORT
1	SAILING	1	20	1	3
2	SAILING	2	10	5	2
3	BALLAST	3	5	1	3

INITIAL STATES

PORTS

IPORT	STATUS	IN	COMING	SNUMB	CNUMB
1	NOTFULL	-0	-0	-0	-0
2	NOTFULL	-0	1	-0	-0
3	NOTFULL	-0	2	-0	-0
4	NOTFULL	-0	-0	-0	-0
5	NOTFULL	-0	-0	-0	-0

INITIAL STATES

CARGOS

CARGO	OPORT	DPORT	TYPE	AMOUNT	EDATE	LDATE	DUEDATE	STATUS	SHIP	PDATE	DDATE
1	1	3	OIL	50000	0	5	40	PICKUP	1	2	-0
2	5	2	OIL	60000	0	6	50	PICKUP	2	5	-0
3	3	1	ORE	60000	5	40	70	NREADY	-0	-0	-0
4	4	3	ORE	60000	10	44	70	NREADY	-0	-0	-0
5	4	2	ORE	60000	50	74	80	NREADY	-0	-0	-0
6	5	4	ORE	60000	5	50	80	NREADY	-0	-0	-0
7	4	3	ORE	60000	12	44	70	NREADY	-0	-0	-0
8	2	4	ORE	62000	5	10	40	NREADY	-0	-0	-0
9	3	1	ORE	61000	5	40	70	NREADY	-0	-0	-0
10	1	4	OIL	60000	42	180	170	NREADY	-0	-0	-0
11	1	5	OIL	60000	60	180	170	NREADY	-0	-0	-0
12	2	4	OIL	61000	120	140	170	NREADY	-0	-0	-0
13	3	1	ORE	61000	100	140	170	NREADY	-0	-0	-0
14	4	2	OIL	61000	100	140	170	NREADY	-0	-0	-0
15	5	3	ORE	60000	110	150	180	NREADY	-0	-0	-0

```
CARGO TOO LARGE  ICARGO=   8  ISHIP=   1
CARGO TOO LARGE  ICARGO=   8  ISHIP=   2
CARGO TOO LARGE  ICARGO=   9  ISHIP=   1
CARGO TOO LARGE  ICARGO=   9  ISHIP=   2
CARGO TOO LARGE  ICARGO=  12  ISHIP=   1
CARGO TOO LARGE  ICARGO=  12  ISHIP=   2
CARGO TOO LARGE  ICARGO=  13  ISHIP=   1
CARGO TOO LARGE  ICARGO=  13  ISHIP=   2
CARGO TOO LARGE  ICARGO=  14  ISHIP=   1
CARGO TOO LARGE  ICARGO=  14  ISHIP=   2
CARGO TOO LARGE  ICARGO=   8  ISHIP=   1
CARGO TOO LARGE  ICARGO=   8  ISHIP=   2
CARGO TOO LARGE  ICARGO=   9  ISHIP=   1
CARGO TOO LARGE  ICARGO=   9  ISHIP=   2
CARGO TOO LARGE  ICARGO=  12  ISHIP=   1
CARGO TOO LARGE  ICARGO=  12  ISHIP=   2
CARGO TOO LARGE  ICARGO=  13  ISHIP=   1
CARGO TOO LARGE  ICARGO=  13  ISHIP=   2
CARGO TOO LARGE  ICARGO=  14  ISHIP=   1
CARGO TOO LARGE  ICARGO=  14  ISHIP=   2
```

POSSIBLE ASSIGNMENTS

	1	2	3
1	1	1	1
2	1	1	1
3	1	1	1
4	1	1	1
5	1	1	1
6	1	1	1
7	1	1	1
8	0	0	1
9	0	0	1
10	1	1	1
11	1	1	1
12	0	0	1
13	0	0	1
14	0	0	1
15	1	1	1

ASSIGNMENT

	1	2	3
1	-1	-0	-0
2	-0	-1	-0
3	-0	-0	-1
4	2	2	2
5	4	5	8
6	3	4	9
7	5	3	7
8	0	0	3
9	0	0	4
10	6	6	11
11	7	7	5
12	0	0	9
13	0	0	7
14	0	0	10
15	8	6	6

10/14/70*SCHEDULE*04 IS NOT FEASABLE CHANGES HAVE BEEN MADE BY THE PROGRAM

SHIP NUMBER 2 ASSIGNMENT NO. 1

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
2	10.0	-0.0	2.0	3.0	-0.0	307900.0	1800000.0	1492100.0

TOTALS TO DATE 15

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
1	10.0	-0.0	2.0	3.0	-0.0	307900.0	1800000.0	1492100.0

SHIP NUMBER 1 ASSIGNMENT NO. 1

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
1	20.0	-0.0	2.0	1.0	-0.0	804000.0	5000000.0	4196000.0

TOTALS TO DATE * 23

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
1	20.0	-0.0	2.0	1.0	-0.0	804000.0	5000000.0	4196000.0

SHIP NUMBER 3 ASSIGNMENT NO. 1

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
3	17.0	5.0	1.0	1.0	-0.0	750000.0	2400000.0	1650000.0

TOTALS TO DATE 28

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
1	17.0	5.0	3.0	1.0	-0.0	750000.0	2400000.0	1650000.0

SHIP NUMBER 2 ASSIGNMENT NO. 2

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
4	15.0	24.0	1.0	1.0	0.0	1053200.0	3000000.0	1946800.0

TOTALS TO DATE 60

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
2	25.0	24.0	4.0	4.0	-0.0	1361100.0	4800000.0	3438900.0

SHIP NUMBER 3 ASSIGNMENT NO. 3

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
8	13.0	16.0	3.0	1.0	0.0	970000.0	3100000.0	2130000.0

TOTALS TO DATE 65

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
2	30.0	21.0	8.0	2.0	-0.0	1720000.0	5500000.0	3780000.0

SHIP NUMBER 1 ASSIGNMENT NO. 3

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
6	20.0	25.0	3.0	1.0	0.0	1555500.0	3000000.0	1444500.0

TOTALS TO DATE 76

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
2	40.0	25.0	8.0	2.0	-0.0	2359500.0	8000000.0	5640500.0

SHIP NUMBER 1 ASSIGNMENT NO. 4

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
5	20.0	0.0	1.0	3.0	0.0	827500.0	6000000.0	5172500.0

TOTALS TO DATE 103

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
3	60.0	25.0	10.0	5.0	-0.0	3187000.0	14000000.0	10813000.0

SHIP NUMBER 2 ASSIGNMENT NO. 3

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
7	15.0	24.0	1.0	1.0	0.0	1088200.0	3000000.0	1911800.0

TOTALS TO DATE 105

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
3	40.0	48.0	8.0	5.0	-0.0	2449300.0	7800000.0	5350700.0

SHIP NUMBER 3 ASSIGNMENT NO. 4

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
9	17.0	20.0	1.0	1.0	0.0	1238000.0	2440000.0	1202000.0

TOTALS TO DATE 108

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
3	47.0	41.0	10.0	3.0	-0.0	2958000.0	7940000.0	4982000.0

SHIP NUMBER 2 ASSIGNMENT NO. 6

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
15	12.0	20.0	3.0	1.0	0.0	866600.0	3000000.0	2133400.0

TOTALS TO DATE 145

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
4	52.0	68.0	14.0	6.0	-0.0	3315900.0	10800000.0	7484100.0

SHIP NUMBER 3 ASSIGNMENT NO. 5

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
11	33.0	0.0	1.0	3.0	0.0	1165000.0	2400000.0	1235000.0

TOTALS TO DATE 148

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
4	80.0	41.0	13.0	6.0	-0.0	4123000.0	10340000.0	6217000.0

SHIP NUMBER 1 ASSIGNMENT NO. 6

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
10	33.0	25.0	1.0	1.0	0.0	2072500.0	3000000.0	927500.0

TOTALS TO DATE 167

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
4	93.0	50.0	12.0	6.0	-0.0	5259500.0	17000000.0	11740500.0

SHIP NUMBER 3 ASSIGNMENT NO. 7

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
13	17.0	16.0	1.0	1.0	0.0	1097000.0	2440000.0	1343000.0

TOTALS TO DATE 187

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
5	97.0	57.0	15.0	7.0	-0.0	5220000.0	12780000.0	7560000.0

SHIP NUMBER 3 ASSIGNMENT NO. 9

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
12	13.0	16.0	3.0	1.0	0.0	999000.0	3050000.0	2051000.0

TOTALS TO DATE 224

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
6	110.0	73.0	20.0	8.0	-0.0	6219000.0	15830000.0	9611000.0

SHIP NUMBER 3 ASSIGNMENT NO. 10

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
14	13.0	0.0	1.0	3.0	0.0	539000.0	1830000.0	1291000.0

TOTALS TO DATE 244

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
7	123.0	73.0	22.0	11.0	-0.0	6758000.0	17660000.0	10902000.0

****SCHEDULE COMPLETED ON DATE 244

ASSIGNMENT STATUS

	1	2	3
1	-1	-0	-0
2	-0	-1	-0
3	-0	-0	-1
4	0	-1	0
5	-1	0	0
6	-1	0	0
7	0	-1	0
8	0	0	-1
9	0	0	-1
10	-1	0	0
11	0	0	-1
12	0	0	-1
13	0	0	-1
14	0	0	-1
15	0	-1	0

FINAL STATES

SHIPS

ISHIP	ACTION	CARGO	RELTIME	OPORT	DPORT
1	FREE	0	167	4	0
2	FREE	0	145	3	0
3	FREE	0	244	2	0

FINAL STATES

PORTS

IPORT	STATUS	IN	COMING	SNUMB	CNUMB
1	NOTFULL	0	0	-0	0
2	NOTFULL	0	0	-0	0
3	NOTFULL	0	0	-0	0
4	NOTFULL	0	0	-0	0
5	NOTFULL	0	0	-0	0

TOTALS BY SHIP

SHIP	CARGOS	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
1	4.0	93.0	50.0	12.0	6.0	-0.0	5259500.0	17000000.0	11740500.0
2	4.0	52.0	68.0	14.0	6.0	-0.0	3315900.0	10800000.0	7484100.0
3	7.0	123.0	73.0	22.0	11.0	-0.0	6758000.0	17660000.0	10902000.0

GRAND TOTALS

CARGO	SAILING	BALLAST	LOADING	UNLOADING	WAITING	COST	BENEFITS	PROFIT
15	268.0	191.0	48.0	23.0	0.0	15333400.0	45460000.0	30126600.0

FINAL STATES

CARGOS

CARGO	OPORT	DPORT	TYPE	AMOUNT	EDATE	LDATE	DUEDATE	STATUS	SHIP	PDATE	DDATE
1	1	3	OIL	50000	0	5	40	PICKUP	1	2	23
2	5	2	OIL	60000	0	6	50	PICKUP	2	5	15
3	3	1	ORE	60000	5	40	70	PICKUP	3	6	28
4	4	3	ORE	60000	10	44	70	PICKUP	2	40	60
5	4	2	ORE	60000	50	74	80	PICKUP	1	78	103***LATE***
6	5	4	ORE	60000	5	50	80	PICKUP	1	49	76
7	4	3	ORE	60000	12	44	70	PICKUP	2	85	105***LATE***
8	2	4	ORE	62000	5	10	40	PICKUP	3	45	65***LATE***
9	3	1	ORE	61000	5	40	70	PICKUP	3	86	108***LATE***
10	1	4	OIL	60000	42	180	170	PICKUP	1	129	167
11	1	5	OIL	60000	60	180	170	PICKUP	3	110	148
12	2	4	OIL	61000	120	140	170	PICKUP	3	204	224***LATE***
13	3	1	ORE	61000	100	140	170	PICKUP	3	165	187***LATE***
14	4	2	OIL	61000	100	140	170	PICKUP	3	226	244***LATE***
15	5	3	ORE	60000	110	150	180	PICKUP	2	126	145

APPENDIX B

SCHEDULES THAT WERE EVALUATED

A MARITIME TRANSPORTATION NETWORK
10/12/70* SCHEDULE* 01

ASSIGNMENT

	1	2	3
1	-1	-0	-0
2	-0	-1	-0
3	-0	-0	-1
4	2	2	2
5	4	5	8
6	3	4	9
7	5	3	7
8	0	0	3
9	0	0	4
10	6	6	11
11	7	7	10
12	0	0	7
13	0	0	7
14	0	0	6
15	8	6	9

A MARITIME TRANSPORTATION NETWORK
10/12/70* SCHEDULE* 02

ASSIGNMENT

	1	2	3
1	-1	-0	-0
2	-0	-1	-0
3	-0	-0	-1
4	2	2	2
5	4	5	8
6	3	4	9
7	5	3	7
8	0	0	3
9	0	0	4
10	6	6	11
11	7	7	5
12	0	0	9
13	0	0	7
14	0	0	10
15	8	6	6

A MARITIME TRANSPORTATION NETWORK
10/13/70* SCHEDULE* 03

ASSIGNMENT

	1	2	3
1	-1	-0	-0
2	-0	-1	-0
3	-0	-0	-1
4	2	2	2
5	4	5	8
6	2	4	9
7	5	3	7
8	0	0	3
9	0	0	4
10	3	6	11
11	7	7	5
12	0	0	9
13	0	0	7
14	0	0	10
15	8	6	6

A MARITIME TRANSPORTATION NETWORK
10/14/70* SCHEDULE* 04

ASSIGNMENT

	1	2	3
1	-1	-0	-0
2	-0	-1	-0
3	-0	-0	-1
4	2	2	2
5	4	5	8
6	3	4	9
7	5	3	7
8	0	0	3
9	0	0	4
10	6	6	11
11	7	7	5
12	0	0	9
13	0	0	7
14	0	0	10
15	8	6	6

A MARITIME TRANSPORTATION NETWORK
10/14/70* SCHEDULE* 05

ASSIGNMENT

	1	2	3
1	-1	-0	-0
2	-0	-1	-0
3	-0	-0	-1
4	2	2	2
5	4	5	8
6	2	4	9
7	5	3	7
8	0	0	3
9	0	0	4
10	6	-0	-0
11	-0	7	-0
12	0	0	9
13	0	0	7
14	0	0	10
15	8	6	6

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